Answer on Question #53657 – Math – Statistics and Probability

The Wechsler IQ is designed so that the means is 100 and the standard deviation is 15 for the population of normal adults. Listed below are IQ scores for professional pilots. It is claimed that because professional pilots are a more homogeneous group than the general population, they have IQ scores with a standard deviation less than 15. Test that claim using a .05 significance level. 121 116 115 121 116 107 127 98 116 101 130 114

Solution

In given problem we need to formulate the null and the alternative hypothesis. Every hypothesis test requires the analyst to state a null hypothesis and an alternative hypothesis. The hypotheses are stated in such a way that they are mutually exclusive. That is, if one is true, the other must be false; and vice versa.

We have to verify with a reliability of 95% that the standard deviation is less than 15.

First, based on the given data, we can determine the average level IQ scores for 12 professional pilots.

$$\text{Mean} = \frac{\sum x_i}{n}$$

$$= \frac{121 + 116 + 115 + 121 + 116 + 107 + 127 + 98 + 116 + 101 + 130 + 114}{12} = \frac{1382}{12}$$

$$= 115.167$$

Then, we can calculate the value of Standard deviation. We apply the following formula.

$$\text{Standard deviation}$$

$$= \sqrt{\frac{(121 - 115.167)^2 + (116 - 115.167)^2 + (115 - 115.167)^2 + (121 - 115.167)^2 + (116 - 115.167)^2 + (107 - 115.167)^2}{12 - 1}}$$

$$= \sqrt{\frac{(127 - 115.167)^2 + (98 - 115.167)^2 + (116 - 115.167)^2 + (101 - 115.167)^2 + (130 - 115.167)^2 + (114 - 115.167)^2}{12 - 1}}$$

$$= \sqrt{\frac{993.6666667}{11}} = \sqrt{90.3333} \approx 9.504384$$

Thus, the sample standard deviation is equal to 9.504.

Now, we need to state the null and alternative hypothesis.

Null hypothesis H_0: $s \geq 15$ (Professional pilots have IQ scores with a standard deviation at least 15)

Alternative hypothesis H_a: $s < 15$ (Professional pilots have IQ scores with a standard deviation less than 15)

We note that these hypotheses constitute a one-tailed test.

Significance level is $\alpha = 0.05$

Then we use the test statistic:

$$\chi^2 = \frac{(n-1)S_n^2}{\sigma_0^2} \leq \chi_\alpha^2(n-1)$$

We substitute the values into the formula.

$$\chi^2 = \frac{(n-1)S_n^2}{\sigma_0^2} = \frac{(12-1)(9.504)^2}{(15)^2} = \frac{993.6666667}{225} = 4.4163$$

The degrees of freedom (DF) are equal to

$$df = (12-1)(2-1) = 11$$

Then, from the Chi-Square Distribution Table we find the critical value (http://sites.stat.psu.edu/~mga/401/tables/Chi-square-table.pdf), $\chi^2_{0.950} = 4.575.$, when df=11.

We have found that the critical value $4.575 > 4.4163$, thus, we have to reject the null hypothesis, which states that the professional pilots have IQ scores with a standard deviation at least 15 and we have enough evidence to support the claim that they have IQ scores with a standard deviation less than 15.