Answer on Question #53552 – Math – Statistics and Probability

In an effort to estimate the annual income for new graduates, data were collected from 100 new graduates over a 1-year period. Assume a population standard deviation of $2,000.

If the sample mean is $15,000, what is the 95% confidence interval for the population mean?

a. $14,590 to $15,410
b. $14,668 to $15,332
c. $14,608 to $15,392
d. $14,603 to $15,396
e. $14,534 to $15,466

If the sample mean is $15,000, what is the 98% confidence interval for the population mean?

a. $14,668 to $15,332
b. $14,608 to $15,392
c. $14,603 to $15,396
d. $14,534 to $15,466
e. $14,590 to $15,410

Solution

The confidence interval does not reflect the variability in the unknown parameter. Rather, it reflects the amount of random error in the sample and provides a range of values that are likely to include the unknown parameter. A confidence interval is the range of likely values of the parameter (defined as the point estimate + margin of error) with a specified level of confidence (which is similar to a probability).

1. If we want to generate a 95% confidence interval estimate for an unknown population mean, this means that there is a 95% probability that the confidence interval will contain the true population mean. Thus, P([sample mean] - margin of error < μ < [sample mean] + margin of error) = 0.95.

In given problem we need to consider the 95% confidence interval for the population mean. Since we are trying to estimate a population mean, we choose the sample mean ($15,000) as the sample statistic. Then, we have to find standard deviation or standard error. Since we do not know the standard deviation of the population, we cannot compute the standard deviation of the sample mean; instead, we compute the standard error (SE). Because the sample size is much smaller than the population size, we can use the "approximate" formula for the standard error.

Standard deviation (also called the standard error):

$$\text{Standard error} = \sigma_x = \frac{\sigma}{\sqrt{n}} = \frac{\$2000}{\sqrt{100}} = \frac{\$2000}{10} = \$200$$

Then we need to find critical value. The critical value is a factor used to compute the margin of error. For this example, we will express the critical value as a t-score. To find the critical value we compute alpha (α):

$$\alpha = 1 - \frac{(\text{confidence level})}{100} = 1 - \frac{95}{100} = 1 - 0.95 = 0.05$$

Next we calculate the critical probability p:

$$p = 1 - \frac{\alpha}{2} = 1 - \frac{0.05}{2} = 1 - 0.025 = 0.975$$

We find the degrees of freedom (df): $df = n - 1 = 100 - 1 = 99$

The critical value is the t-score having 99 degrees of freedom and a cumulative probability equal to 0.975. From the table Critical values of Student's t distribution with n degrees of freedom, we determine the critical value, which is equal to 1.984.

Compute margin of error (ME) for 95% confidence, we have

$$\bar{x} \pm 1.984 \cdot \text{standard error}$$

Substituting the values $n = 100, \sigma = \$2000, \bar{x} = \$15000$ into the formula we obtain

$$\$15000 \pm 1.984 \cdot (\$200)$$

$$\$15000 \pm 396.8$$

We might also have expressed the critical value as a z score. Because the sample size is fairly large, a z score analysis produces a similar result - a critical value is equal to 1.96.

Substituting the sample statistics and the Z value for 95% confidence, we have

$$\$15000 \pm 1.96 \cdot (\$200)$$

$$\$15000 \pm 392$$

Thus, we are 95% confident that the true mean is between $14 608 and $15 392.

The answer is c. $14,608 to $15,392

2. Now, we need to determine the 98% confidence interval for the population mean of $15000.

We apply the same method of calculation, but we have to recalculate the critical value.

First, we find alpha (α):

$$\alpha = 1 - \frac{(\text{confidence level})}{100} = 1 - \frac{98}{100} = 1 - 0.98 = 0.02$$

Next we calculate the critical probability p:

$$p = 1 - \frac{\alpha}{2} = 1 - \frac{0.02}{2} = 1 - 0.01 = 0.99$$

We calculate the degrees of freedom (df): $df = n - 1 = 100 - 1 = 99$

The critical value is the t-score having 99 degrees of freedom and a cumulative probability equal to 0.99. From the table Critical values of Student's t distribution with n degrees of freedom, we determine the critical value, which is equal to 2.365.

Now we can compute margin of error (ME) for 98% confidence, we have

$$\bar{x} \pm 2.365 \cdot \text{standard error}$$

Substituting the values $n = 100, \sigma = \$2000, \bar{x} = \$15000$ into the formula we obtain

$$\$15000 \pm 2.365 \cdot (\$200)$$

$$\$15000 \pm 473$$

We might also have expressed the critical value as a z-score. Because the sample size is fairly large, a z-score analysis produces a similar result - a critical value is equal to 2.33.

Then, the 98% confidence interval for the population mean will be equal to

$$\$15000 \pm 2.33 \cdot (\$200)$$

$$\$15000 \pm 466$$

Therefore, the 98% confidence interval is $14 534 to $15 466. That is, we are 98% confident that the true population mean is in the range defined by $\$15000 \pm 466$.

The answer is d. $14,534 to $15,466.