Answer on Question #53549 – Math – Statistics and Probability

If a simple random sample of 200 provides 50 'yes' responses, the 89% confidence interval for the population proportion is

a. 49.951 to 199.951.

b. .201 to .299.

c. 199.951 to 200.049.

d. 44.4 to 55.6.

Solution

In order to construct a confidence interval for a sample proportion, we need to know the variability of the sample proportion. This means we need to know how to compute the standard deviation and/or the standard error of the sampling distribution.

Let p denote the population proportion. To estimate p, we form a sample and the sample proportion which we will call $\hat{p}$.

There are 50 respondents provided the answer “yes’, so the sample proportion is

$$\hat{p} = \frac{X}{n} = \frac{50}{200} = 0.25$$

Point estimate is a single value used to approximate a population parameter. The sample proportion $\hat{p}$ is the best point estimate of the population proportion p. Sample proportion of x successes in a sample of size n. It is an unbiased estimate (best estimate).

$$\hat{q} = 1 - \hat{p} = 1 - 0.25 = 0.75$$

A sample proportion of failures in a sample size of n. Proportions of population is found using confidence intervals and by using the sample proportion $\hat{p}$.

The standard error of proportion in the sample is given by

Standard error (p) = $\sqrt{\frac{\hat{p}\hat{q}}{n}}$

Now, we compute the standard error.

$$SE = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.25(1-0.25)}{200}} = 0.030618$$

Then, we need to find the critical value. The critical value is a factor used to compute the margin of error. Because the sampling distribution is approximately normal and the sample size is large, we can express the critical value as a z score.

First, we calculate alpha (α):

$$\alpha = 1 - \frac{(\text{confidence level})}{100} = 1 - \frac{89}{100} = 1 - 0.89 = 0.11$$

Then, we find the critical probability:

$$p = 1 - \frac{\alpha}{2} = 1 - \frac{0.11}{2} = 1 - 0.055 = 0.945$$

Find the degrees of freedom (df): df = n - 1 = 200 -1 = 199

Next, we define the critical value. We will express the critical value as a t-score. For this problem, it will be the t score having 199 degrees of freedom and a cumulative probability equal to 0.945. Using the table of Critical values of Student's t distribution with n degrees of freedom we find that the critical value, which is equal to 1.605.

Finally, we can determine the margin of error:

$$\text{Margin of error} = \text{critical value} \cdot \text{standard error} = 1.605 \cdot 0.030618 = 0.049143$$

Thus, the 89% confidence limits for population proportion P are given by

$$0.25 \pm 0.0491$$

Therefore, the 89% confidence interval is 0.201 to 0.299. That is, the 89% confidence interval is the range defined by 0.25 ± 0.0491.

Thus, the answer is b. .201 to .299.

Answer: b. .201 to .299.