Answer on Question #53547 – Math – Statistics and Probability

Sample 1: n1 = 50, sample mean = 22, Standard deviation = 4

Sample 2: n2 = 35, sample mean = 14, Standard deviation = 5

The 90% confidence interval for the difference between the two population means is?

The 97% confidence interval for the difference between the two population means is?

Solution

We have the following given data: $n_1 = 50, n_2 = 35, \bar{x}_1 = 22, \bar{x}_2 = 14, s_1 = 4, s_2 = 5$.

A confidence interval gives an estimated range of values which is likely to include an unknown population parameter, the estimated range being calculated from a given set of sample data. Assume that the two populations are independent and normally distributed.

1. First, we are working with a **90% confidence interval**.

Since we are trying to estimate the difference between population means, we choose the difference between sample means as the sample statistic. Thus,

$$\bar{x}_1 - \bar{x}_2 = 22 - 14 = 8$$

Next, we compute the standard error (SE), which is an estimate of the standard deviation of the difference between sample means by using the sample standard deviations.

$$\text{Standard error} = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

We substitute the given values into the noted above formula.

$$\text{Standard error} = \sqrt{\frac{(4)^2}{50} + \frac{(5)^2}{35}} = \sqrt{0.32 + 0.7143} = \sqrt{1.034285714} = 1.017$$

If we are working with a pooled standard deviation, then the degree of freedom:

$$\text{DF} = n_1 + n_2 - 2$$

$$\text{DF} = 50 + 35 - 2 = 83$$

Now, we need to determine the critical value. The critical value is a factor used to compute the margin of error. Because the sample sizes are large enough, we express the critical value as a z-score.

First, we calculate alpha (α):

$$\alpha = 1 - \frac{(\text{confidence level})}{100} = 1 - \frac{90}{100} = 1 - 0.9 = 0.1$$

Then, we find the critical probability:

$$p = 1 - \frac{\alpha}{2} = 1 - \frac{0.1}{2} = 1 - 0.05 = 0.95$$

The critical value is the z-score having a cumulative probability equal to 0.95.

From the Standard normal distribution table, we have found that the critical value is equal to 1.645.

Finally, we can determine the margin of error:

$$\text{Margin of error} = \text{critical value} \cdot \text{standard error} = 1.645 \cdot 1.017 = 1.67296$$

The interval estimator is

$$(\bar{\mathrm{x}}_1 - \bar{\mathrm{x}}_2) \pm z_\alpha \cdot S_{\bar{\mathrm{x}}_1 - \bar{\mathrm{x}}_2}$$

Now, we can substitute into the formula the margin of error in order to find the 90% confidence interval for the difference between the two population means.

$$8 \pm 1.645 \cdot 1.017$$

$$8 \pm 1.67296$$

Therefore, the 90% confidence interval is (6.3270 ; 9.6730).

2. Now, our task is to determine the **97% confidence interval** for the difference between the two population means.

We have already found the Standard error, which in this case is the same.

We need to determine the critical value based on the percentage for the confidence level (97% in this case).

We calculate alpha (α):

$$\alpha = 1 - \frac{(\text{confidence level})}{100} = 1 - \frac{97}{100} = 1 - 0.97 = 0.03$$

Then, we find the critical probability:

$$p = 1 - \frac{\alpha}{2} = 1 - \frac{0.03}{2} = 1 - 0.015 = 0.985$$

The critical value is the z-score having a cumulative probability equal to 0.985.

From the Standard normal distribution table, we have found that the critical value is equal to 2.170.

Finally, we can determine the margin of error:

$$\text{Margin of error} = \text{critical value} \cdot \text{standard error} = 2.17 \cdot 1.017 = 2.2069$$

The confidence interval formula is

$$(\bar{x}_1 - \bar{x}_2) \pm z_\alpha \cdot S_{\bar{x}_1 - \bar{x}_2}$$

Now, we can substitute the margin of error into the formula in order to find the 97% confidence interval for the difference between the two population means:

$$8 \pm 2.17 \cdot 1.017$$

$$8 \pm 2.2069$$

Therefore, the 97% confidence interval is (5.7931; 10.2069).