**Answer on Question #53441 – Math –Statistics and Probability**

The joint distribution of two variables X and Y is given below.

Table 1

| | Y | |
|---|---|---|
| | 0 | 1 |
| X | | |
| 0 | 0.1 | 0.2 |
| 1 | 0.4 | 0.2 |
| 2 | 0.1 | 0 |

Evaluate the following:

**(i)** Marginal distribution of X and Y;
**(ii)** E(XY)
**(iii)** COV(X,Y),
**(iv)** P(X + Y>1),
**(v)** verify that X and Y are not independent.

**Solution**

**(i)** X can take on values 0, 1 and 2.

Evaluate

$$P(X=0)=P(X=0,Y=0)+P(X=0,Y=1)=0.1+0.2=0.3$$

$$P(X=1)=P(X=1,Y=0)+P(X=1,Y=1)=0.4+0.2=0.6$$

$$P(X=2)=P(X=2,Y=0)+P(X=2,Y=1)=0.1+0=0.1$$

Y can take on values 0 and 1.

Evaluate

$$P(Y=0)=P(X=0,Y=0)+P(X=1,Y=0)+P(X=2,Y=0)=0.1+0.4+0.1=0.6$$

$$P(Y=1)=P(X=0,Y=1)+P(X=1,Y=1)+P(X=2,Y=1)=0.2+0.2+0=0.4$$

So marginal distribution of X is

Table 2

| X=$x_i$ | 0 | 1 | 2 |
|---|---|---|---|
| P(X=$x_i$) | 0.3 | 0.6 | 0.1 |

So marginal distribution of Y is

Table 3

| Y=$y_i$ | 0 | 1 |
|---|---|---|
| P(Y=$y_i$) | 0.6 | 0.4 |

**(ii)** XY can take on values 0, 1, 2.
Evaluate

$P(XY=0)=P(X=0,Y=0)+P(X=0,Y=1)+P(X=1,Y=0)+P(X=2,Y=0)$
$=0.1+0.2+0.4+0.1=0.8$

$P(XY=1)=P(X=1,Y=1)=0.2$

$P(XY=2)=P(X=2,Y=1)=0$

E(XY)= $\sum t\cdot P(XY=t)=0\cdot 0.8+1\cdot 0.2+2\cdot 0=0.2$

**(iii)** Compute

E(X)= $\sum t\cdot P(X=t)=0\cdot 0.3+1\cdot 0.6+2\cdot 0.1=0.8$

E(Y)= $\sum t\cdot P(Y=t)=0\cdot 0.6+1\cdot 0.4=0.4$

COV(X,Y)=E(XY)-EX·EY=0.2-0.8· 0.4=0.2-0.32= -0.12

**(iv)** X+Y can take on values 0,1, 2, 3

P(X + Y>1)= P(X + Y=2)+ P(X + Y=3)= P(X =2, Y=0)+ P(X =1, Y=1)+

+ P(X =2, Y=1)=0.1+0.2+0=0.3.

**(v)** If X and Y are independent, then E(XY)=E(X)·E(Y), but in this problem that rule breaks, because E(XY)=0.2≠ 0.32=E(X)·E(Y). Thus, X and Y are not independent.