Answer on Question #53406 – Math – Calculus

A geometric series has first term a and common ratio r where a and r are positive constants.The fourth term of the series is 1/p , the sum to infinity of the series is p and the sum of the first two terms of the series is 0.75p where p is a positive constant.
Find the values of a,r and p.

Solution

$$a_1 = a,\ \ a_4 = ar^3 = \frac{1}{p},\ \ S_2 = a_1 + a_2 = a + ar = 0.75p,\ S = \frac{a}{1-r} = p.$$

So **adding** $1 + r = \frac{3}{4}\frac{p}{a}$ **and** $1 - r = \frac{a}{p} \rightarrow\ 2 = \frac{3p}{4a} + \frac{a}{p} \rightarrow$

$$\rightarrow \frac{3}{4}\left(\frac{p}{a}\right)^2 - 2\frac{p}{a} + 1 = 0 \rightarrow$$

$\rightarrow \frac{p}{a} = 2\ \ or\ \frac{p}{2} = \frac{2}{3}.$

1) $\frac{p}{a} = 2\ \rightarrow\ 1 + r = \frac{3}{4} * 2 = \frac{3}{2}\ \rightarrow\ r = \frac{1}{2};$

$ar^3 = \frac{1}{p} \rightarrow\ p = \frac{8}{a},\ \frac{a}{1-r} = p\ \rightarrow\ 2a = p\ \rightarrow\ 2a = \frac{8}{a}\ \rightarrow\ a = 2,\ p = 4.$

2) $\frac{p}{a} = \frac{2}{3} \rightarrow\ 1 + r = \frac{3}{4} * \frac{2}{3} = \frac{1}{2}\ \rightarrow\ r = -\frac{1}{2} < 0$**, which contradicts the requirement r is a positive constant. Therefore case** $\frac{p}{a} = \frac{2}{3}$ **is impossible.**

Thus we have the only solution: $a = 2,\ r = \frac{1}{2},\ p = 4.$