Answer on Question #53387 – Math –Statistics and Probability

The joint distribution of two variables X and Y is given below.

Table 1

	Y	
	0	1
X		
0	0.1	0.2
1	0.4	0.2
2	0.1	0

Evaluate the following:

(i) Marginal distribution of X and Y;
(ii) E(XY)
(iii) COV(X,Y),
(iv) P(X + Y>1),
(v) verify that X and Y are not independent.

Solution

(i) X can take on values 0, 1 and 2.

Evaluate

$$P(X=0)=P(X=0,Y=0)+P(X=0,Y=1)=0.1+0.2=0.3$$

$$P(X=1)=P(X=1,Y=0)+P(X=1,Y=1)=0.4+0.2=0.6$$

$$P(X=2)=P(X=2,Y=0)+P(X=2,Y=1)=0.1+0=0.1$$

Y can take on values 0 and 1.

Evaluate

$$P(Y=0)=P(X=0,Y=0)+P(X=1,Y=0)+P(X=2,Y=0)=0.1+0.4+0.1=0.6$$

$$P(Y=1)=P(X=0,Y=1)+P(X=1,Y=1)+P(X=2,Y=1)=0.2+0.2+0=0.4$$

So marginal distribution of X is

Table 2

X=x_i	0	1	2
P(X=x_i)	0.3	0.6	0.1

So marginal distribution of Y is

Table 3

Y=y_i	0	1
P(Y=y_i)	0.6	0.4

(ii) XY can take on values 0, 1, 2.
Evaluate

$$P(XY = 0) = P(X = 0, Y = 0) + P(X = 0, Y = 1) + P(X = 1, Y = 0) + P(X = 2, Y = 0)$$
$$= 0.1 + 0.2 + 0.4 + 0.1 = 0.8$$

$$P(XY = 1) = P(X = 1, Y = 1) = 0.2$$

$$P(XY = 2) = P(X = 2, Y = 1) = 0$$

E(XY)= $\sum t \cdot P(XY = t) = 0 \cdot 0.8 + 1 \cdot 0.2 + 2 \cdot 0 = 0.2$

(iii) Compute

E(X)= $\sum t \cdot P(X = t) = 0 \cdot 0.3 + 1 \cdot 0.6 + 2 \cdot 0.1 = 0.8$

E(Y)= $\sum t \cdot P(Y = t) = 0 \cdot 0.6 + 1 \cdot 0.4 = 0.4$

COV(X,Y)=E(XY)-EX·EY=0.2-0.8· 0.4=0.2-0.32= -0.12

(iv) X+Y can take on values 0,1, 2, 3

P(X + Y>1)= P(X + Y=2)+ P(X + Y=3)= P(X =2, Y=0)+ P(X =1, Y=1)+

+ P(X =2, Y=1)=0.1+0.2+0=0.3.

(v) If X and Y are independent, then E(XY)=E(X)·E(Y), but in this problem that rule breaks, because E(XY)=0.2≠ 0.32=E(X)·E(Y). Thus, X and Y are not independent.