Answer on Question #53385 – Math – Statistics and Probability

The following data show systolic blood pressure levels (mm Hg) of a random sample of six patients undergoing a particular drug therapy for hypertension.

182 179 154 161 170 151

Can we conclude, on the basis of these data, that the population mean is greater than 165?

Hint: Use an appropriate parametric test, at the 5% significance level.

Solution:

To begin with, we identify a hypothesis or claim that we feel should be tested. Hypothesis Testing (or significance testing) is a mathematical model for testing a claim, an idea or hypothesis about a parameter of interest in a given population set, using data measured in a sample set. Calculations are performed on selected samples to gather more decisive information about characteristics of the entire population, which enables a systematic way to test claims or ideas about the entire dataset.

The first step is to state the null hypothesis and an alternative hypothesis.

Null hypothesis: $\mu = 165$ (the average level of systolic blood pressure is equal to 165 mm Hg)

Alternative hypothesis: $\mu > 165$ (the average level of systolic blood pressure is greater than 165 mm Hg)

We note that these hypotheses constitute a one-tailed test. The null hypothesis will be rejected if the sample mean is too small.

Set the criteria for a decision. The level of significance is 0.05, which makes the alpha level α= 0.05. To determine the critical value for an upper-tail critical test, we locate the probability .0500 toward the tail in column C in the unit normal table. From the given data, we can determine the average level of the systolic blood pressure based on the random sample of six patients.

Mean = $\frac{\sum x}{n} = \frac{182+179+154+161+170+151}{6} = 166.167\ (mm\ Hg)$

Degrees of freedom (DF) = n - 1 = 6 - 1 = 5

$$Standard\ deviation = \sqrt{\frac{(182-166.17)^2+(179-166.17)^2+(154-166.17)^2+(161-166.17)^2+(170-166.17)^2+(151-166.167)^2}{6-1}}$$

$$= \sqrt{\frac{834.833}{5}} = \sqrt{166.967} \approx 12.922$$

Critical value is $t_{crit} = 2.015$

The test statistic for examining hypotheses about one population mean:

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

where $\bar{x}$ is the observed sample mean, μ_0 is value specified in then null hypothesis, σ is the standard deviation of the sample measurements and n is the number of differences.

$$t = \frac{166.167 - 165}{\frac{12.922}{\sqrt{6}}} = \frac{1.167}{5.275} = 0.221$$

To make a decision, we compare the obtained value to the critical value. We reject the null hypothesis if the obtained value exceeds the critical value. We define the value of t is less than the critical value.

The p-value for this test is 0.417. Thus, since the P-value (0.417) is greater than the significance level (0.05), we cannot reject the null hypothesis. So, we can't conclude on the basis of these data that the population mean is greater than 165.