The function t(x) at the point T(3,18) has gradient 1.5*t(x). Given the second derivative of the function t(x) is 2(3x + 1), sketch the curve of y=t(x) clearly showing any points of intersection with the coordinate axes.

Solution

$$t''(x) = 2(3x+1) \rightarrow t'(x) = 3x^{2} + 2x + a \rightarrow t(x) = x^{3} + x^{2} + ax + b;$$

$$t(3) = 18 \rightarrow 27 + 9 + 3a + b = 18 \rightarrow 3a + b = -18;$$

$$t'(3) = 1.5t(3) \rightarrow 27 + 6 + a = 1.5(27 + 9 + 3a + b) \rightarrow$$

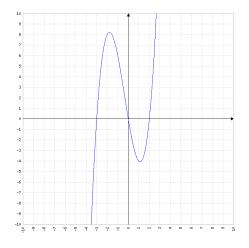
$$\rightarrow 3.5a + 1.5b = -21.$$

So $\begin{cases} 3a + b = -18\\ 7a + 3b = -42 \end{cases} \rightarrow a = -6, b = 0.$
Thus $t(x) = x^{3} + x^{2} - 6x; t(0) = 0^{3} + 0^{2} - 6 \cdot 0 = 0.$

$$x^{3} + x^{2} - 6x = 0 \rightarrow x = 0, \quad x = -3, \quad x = 2.$$

x - intercepts (points of intersection with Ox coordinate axis): (0,0), (-3,0), (2,0);

y – intercept (point of intersection with Oy coordinate axis): (0, 0).



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