Answer on Question #53274 – Math – Calculus

The function t(x) at the point T(3,18) has gradient 1.5*t(x) . Given the second derivative of the function t(x) is 2(3x +1) , sketch the curve of y=t(x) clearly showing any points of intersection with the coordinate axes.

Solution

$$t''(x) = 2(3x+1) \rightarrow \quad t'(x) = 3x^2 + 2x + a \rightarrow \quad t(x) = x^3 + x^2 + ax + b;$$

$$t(3) = 18 \rightarrow \quad 27 + 9 + 3a + b = 18 \rightarrow \quad 3a + b = -18;$$

$$t'(3) = 1.5t(3) \rightarrow \quad 27 + 6 + a = 1.5(27 + 9 + 3a + b) \rightarrow$$

$$\rightarrow \quad 3.5a + 1.5b = -21.$$

So $\begin{cases} 3a + b = -18 \\ 7a + 3b = -42 \end{cases} \rightarrow \quad a = -6, \quad b = 0.$

Thus $t(x) = x^3 + x^2 - 6x;\ t(0) = 0^3 + 0^2 - 6 \cdot 0 = 0.$

$x^3 + x^2 - 6x = 0 \rightarrow \quad x = 0, \qquad x = -3, \qquad x = 2.$

x – intercepts (points of intersection with Ox coordinate axis): $(0,0)$, $(-3,0)$, $(2,0)$;

y – intercept (point of intersection with Oy coordinate axis): $(0,0)$.