Answer on Question #53170 – Math – Statistics and Probability

A,B,C,D cut a pack of 52 cards successively in the order given. If a person who cuts a spade first receives rs.700, what are their respective expectations?

Solution

The probability of the first spade for the first person is

$$P(A) = \frac{13}{52} = 0.25.$$

The respective expectation for the first person is

$$E(A) = x \cdot P(x) = 700 \cdot 0.25 = 175.$$

The probability of the first spade for the second person is

$$P(B) = P\left(B\middle|\overline{A}\right) \cdot P\left(\overline{A}\right) = \frac{13}{51} \cdot \left(1 - \frac{13}{52}\right) = \frac{13 \cdot (52-13)}{51 \cdot 52} = \frac{39}{51 \cdot 4} = \frac{13}{17 \cdot 4}.$$

The respective expectation for the second person is

$$E(B) = x \cdot P(B) = 700 \cdot \frac{13}{17 \cdot 4} \approx 133.82.$$

The probability of the first spade for the third person is

$$P(C) = P\left(B\middle|\overline{A}, \overline{B}\right) \cdot P\left(\overline{A}, \overline{B}\right) = \frac{13}{50} \cdot \cdot \frac{52-13}{52} \cdot \frac{51-13}{51} = \frac{13}{50} \cdot \frac{39}{52} \cdot \frac{38}{51} = \frac{13 \cdot 38}{50 \cdot 4 \cdot 17} = \frac{13 \cdot 19}{17 \cdot 100}.$$

The respective expectation for the third person is

$$E(C) = x \cdot P(B) = 700 \cdot \frac{13 \cdot 19}{17 \cdot 100} \approx 101.71.$$

The probability of the first spade for the fourth person is

$$P(D) = P\left(B\middle|\overline{A}, \overline{B}, \overline{C}\right) \cdot P\left(\overline{A}, \overline{B}, \overline{C}\right) = \frac{13}{49} \cdot \cdot \frac{52-13}{52} \cdot \frac{51-13}{51} \cdot \frac{50-13}{50} = \frac{13}{49} \cdot \frac{39}{52} \cdot \frac{38}{51} \cdot \frac{37}{50} = \frac{13}{49} \cdot \frac{1}{4} \cdot \frac{38}{17} \cdot \frac{37}{50} =$$

$$= \frac{13}{49} \cdot \frac{1}{2} \cdot \frac{19}{17} \cdot \frac{37}{50}.$$

The respective expectation for the fourth person is

$$E(D) = x \cdot P(D) = 700 \cdot \frac{13 \cdot 19 \cdot 37}{49 \cdot 17 \cdot 100} \approx 76.80.$$

Let *F*="a person cuts a spade first", then $F = \begin{cases} 700, p_1 = 1/4 \\ 700, p_2 = 13/(17 \cdot 4) \\ 700, p_3 = (13 \cdot 19)/(17 \cdot 100) \\ 700, p_4 = (13 \cdot 19 \cdot 37)/(49 \cdot 17 \cdot 100) \end{cases}$

The respective expectation is

$$E(F) = 700p_1 + 700p_2 + 700p_3 + 700p_4 = 700 \cdot \left(\frac{1}{4} + \frac{13}{17 \cdot 4} + \frac{13 \cdot 19}{17 \cdot 100} + \frac{13 \cdot 19 \cdot 37}{49 \cdot 17 \cdot 100}\right) = 487.33.$$

Answer: 487.33.