Answer on Question #53016 – Math – Analytic Geometry

Find the distance between the lines $x + 2y = 6$ and $2x + 4y = -9$.

Solution

Method 1

Lines $x + 2y = 6$ and $2x + 4y = -9$ are parallel, because relation between coefficients of two lines

$\frac{1}{2} = \frac{2}{4} \neq \frac{6}{-9}$

holds true.

Rewrite equation of $2x + 4y = -9$ in the following form: $2x + 4y + 9 = 0$, where $a = 2$, $b = 4$, $c = 9$.

Take a point on the first line $x + 2y = 6$, let's say (0,3). Using the formula for distance d from a point (0,3) to line $2x + 4y + 9 = 0$ obtain

$d = \frac{|ax_0+by_0+c|}{\sqrt{a^2+b^2}} = \frac{|2\cdot 0+4\cdot 3+9|}{\sqrt{2^2+4^2}} = \frac{21}{\sqrt{20}} = \frac{21\sqrt{20}}{20} = \frac{21\sqrt{5}}{10} \approx 4.7.$

Method 2

The distance between two lines is defined to be the perpendicular distance between them. The slope of the above two lines is $-\frac{1}{2}$, and the perpendicular line has a slope of 2. The reason is the fact that the product of slopes for two perpendicular lines equals -1.

Thus, perpendicular passing through both lines has an equation $y = 2x + c$.

Take a point on the first line $x + 2y = 6$, let's say (0,3). From this point we can find coefficient c in a perpendicular line $y = 2x + c$ passing thought this point

$$3 = 2 * 0 + c$$

$$c = 3$$

Find the intersection point of perpendicular line $y = 2x + 3$ and line

$2x + 4y = -9$

$$\begin{cases} y = 2x + 3 \\ 2x + 4y = -9 \end{cases}$$

$$\begin{cases} y = 2x + 3 \\ 2x + 4(2x + 3) = -9 \end{cases}$$

$$\begin{cases} y = 2x + 3 \\ 2x + 8x + 12 = -9 \end{cases}$$

$$\begin{cases} y = 2x + 3 \\ 10x = -21 \end{cases}$$

$$\begin{cases} y = 2x + 3 \\ x = -2.1 \end{cases}$$

$$\begin{cases} y = 7.2 \\ x = -2.1 \end{cases}$$

Now, we have two points on both lines ($(0,3)$ *and* $(-2.1,7.2)$). Also both of them lie on the perpendicular line. We can use the formula for the distance between these points:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

$$d = \sqrt{(0 + 2.1)^2 + (3 - 7.2)^2} = \sqrt{4.41 + 17.64} = \sqrt{22.05} \approx \mathbf{4.7}$$

Method 3

We can also choose the other way to find distance between two parallel lines.

If equations of parallel lines are $ax + by + c = 0$ and $ax + by + c_1 = 0$, then the perpendicular distance between them is given by

$$d = \frac{|c - c_1|}{\sqrt{a^2 + b^2}}$$

First rewrite equations of two given lines so that coefficients of x and y are the same in equations of two lines. For given lines it can be done as

$$2x + 4y - 12 = 0$$

$$2x + 4y + 9 = 0$$

Then, using the formula given,

$$d = \frac{|-12 - 9|}{\sqrt{2^2 + 4^2}} = \frac{21}{\sqrt{20}} = \frac{21\sqrt{20}}{20} \approx 4.7$$

Answer: 4.7