**Answer on Question #52873 - Math – Differential Equations**

Solve with the orthogonal trajectory:

x y = c

**Solution**

$$\boldsymbol{xy = c \rightarrow \ \frac{d}{dx}(xy) = \frac{d}{dx}(c) \ \rightarrow \ \ y + x\frac{dy}{dx} = 0 \ \rightarrow \ \frac{dy}{dx} = -\frac{y}{x}.}$$

Using the fact that perpendicular lines have slopes which are negative reciprocals, solve $\boldsymbol{\frac{dY}{dX} = -\frac{1}{\frac{dy}{dx}}}$.

For the orthogonal trajectories

$$\frac{dY}{dX} = \frac{X}{Y}$$

We usually change back to $x$ and $y$ at this point

$$\frac{dy}{dx} = \frac{x}{y}$$

Separate the variables and integrate

$$ydy - xdx = 0$$

$$\frac{y^2}{2} - \frac{x^2}{2} = C_1$$

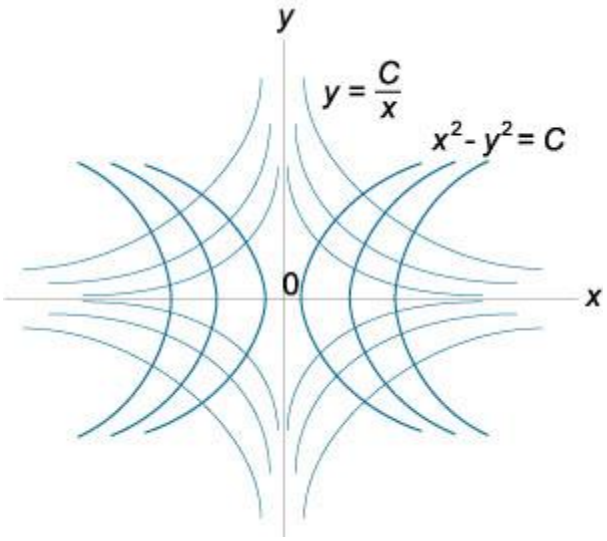
$$y^2 - x^2 = C_2$$

or

$$x^2 - y^2 = C$$

where $C$ is an arbitrary real constant.

Trajectories of x y = c and $x^2 - y^2 = C$ are hyperbolas.


y
$y = \frac{C}{x}$
$x^2 - y^2 = C$
0
x