Answer on Question #52871 – Math – Calculus

Find the Fourier series for the periodic function f(x) with the period 2 π given by

f(x)=2x, - π< x< π

Solution

$$f(x) = \sum_{n=0}^{\infty} A_n cos\left(\frac{n\pi x}{L}\right) + \sum_{n=0}^{\infty} B_n sin\left(\frac{n\pi x}{L}\right)$$

where

$$A_0 = \frac{1}{2L}\int_{-L}^{L} f(x)dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} 2xdx = 0;$$

$$A_n = \frac{1}{L}\int_{-L}^{L} f(x)cos\left(\frac{n\pi x}{L}\right)dx = \frac{1}{\pi}\int_{-\pi}^{\pi} 2xcos\left(\frac{nx}{2}\right)dx = 0;$$

$$B_n = \frac{1}{L}\int_{-L}^{L} f(x)sin\left(\frac{n\pi x}{L}\right)dx = \frac{1}{\pi}\int_{-\pi}^{\pi} 2xsin\left(\frac{nx}{2}\right)dx =$$

$$= \frac{16}{\pi n^2} sin\frac{\pi n}{2} - \frac{8}{n} cos\frac{\pi n}{2} =$$

$$= \begin{cases} \frac{16}{4\pi k^2} sin(\pi k) - \frac{8}{2k} cos(\pi k), n = 2k \\ \frac{16}{\pi(2k+1)^2} sin\left(\frac{\pi}{2} + \pi k\right) - \frac{8}{n} cos\left(\frac{\pi}{2} + \pi k\right), n = 2k+1 \end{cases}$$

$$= \begin{cases} \frac{4}{k}(-1)^{k+1}, n = 2k \\ \frac{16(-1)^k}{\pi(2k+1)^2}, n = 2k+1 \end{cases}$$

So $f(x) = -4\sum_{n=1}^{\infty} \frac{(-1)^n}{n} sin(nx) + \frac{16}{\pi}\sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)^2} sin\left(\frac{2n+1}{2} x\right)$