Answer on Question #52825 – Math – Abstract Algebra

Let N be a normal cyclic subgroup of a group G, then prove that every subgroup of N is normal in G.

Solution

Since H is a subgroup of N, if h is in H, then $h = x^k$ for some integer k. For any g in G, we have $ghg^{-1} = g(x^k)g^{-1} = (gxg^{-1})^k$. Since N is normal, gxg^{-1} is again in N, say $gxg^{-1} = x^m$, for some integer m.

Therefore,

$gxg^{-1} = (gxg^{-1})^k = (x^m)^k = x^{mk} = x^{km} = (x^k)^m = h^m$. Now h^m is in <h>, which is contained in H (H is closed under multiplication), therefore, ghg^{-1} is in H, that is, gHg^{-1} is contained in H, that is, H is normal.