Answer on Question#52763 – Math – Calculus

1. Calculate $\frac{dy}{dx}$ for the following equations:

   a) $cos(x)cos(y) = sin(y)$;

   b) $y = x^{sin\,(x)}$.

   Solution.

   a) $cos(x)cos(y) = sin(y)$. First of all we will express y as function from x:

   $\tan(y) = \cos(x)$. Then $y = \tan^{-1}(\cos(x))$. And now we can calculate $\frac{dy}{dx}$:

   $\frac{dy}{dx} = -\frac{\sin(x)}{1+(\cos(x))^2} = -\frac{\sin(x)}{(\sin(x))^2} = -\frac{1}{\sin(x)}$.

   b) $y = x^{sin\,(x)}$. To find the derivation of this function, we must do next manipulation:

   $\ln y = \ln x^{\sin(x)} = \sin(x)\ln x$. Now we can calculate $\frac{dy}{dx}$:

   $\frac{1}{y}\frac{dy}{dx} = \cos(x)\ln x + \frac{\sin(x)}{x}$. $\frac{dy}{dx} = y\left(\cos(x)\ln x + \frac{\sin(x)}{x}\right. = x^{sin\,(x)}\left(\cos(x)\ln x + \right.$
   $\left. + \frac{\sin(x)}{x}\right)$.

   Answer:

a) $\frac{dy}{dx} = -\frac{1}{\sin(x)}$;

b) $\frac{dy}{dx} = x^{sin\,(x)}\left(\cos(x)\ln x + \frac{\sin(x)}{x}\right)$.