Question #52705– Math – Calculus

Solve the following by Bernoulli equation?

$$\frac{dr}{d\theta} + r\ \tan\theta\ =\ \cos^2\theta\,, r\left(\frac{\pi}{4}\right) = 1$$

Solution:

This is a linear first order ordinary differential equation in the dependent variable x. We write the equation in the form:

$$\frac{dr}{d\theta} + P(\theta)r = Q(\theta)$$

$$\frac{dr}{d\theta} + r\ \tan\theta\ =\ \cos^2\theta$$

So $P(\theta) = \tan\theta\,, Q(\theta) = \cos^2\theta$

The integrating factor is

$$\mu(\theta) = e^{\int P(\theta)d\theta}$$

$$\mu(\theta) = e^{\int \tan\theta d\theta}$$

$$\int \tan\theta\, d\theta = \int \frac{\sin\theta}{\cos\theta} d\theta = \int \frac{-d(\cos\theta)}{\cos\theta} = -\ln\cos\theta$$

$$\mu(\theta) = e^{-\ln\cos\theta} = \frac{1}{\cos\theta}$$

The solution to a linear first order differential equation is then

$$r(\theta) = \frac{\int \mu(\theta)Q(\theta)d\theta + C}{\mu(\theta)}$$

$$r(\theta) = \frac{\int \frac{\cos^2\theta}{\cos\theta} dx + C}{\frac{1}{\cos\theta}} = \sin\theta\cos\theta + C\cos\theta$$

Given $r(\frac{\pi}{4})\ =\ 1$, so

$$1 = \frac{1}{2} + C\frac{1}{\sqrt{2}}$$

$$C = \frac{1}{\sqrt{2}}$$

Answer:

$$\boldsymbol{r = \sin\theta\cos\theta + \frac{1}{\sqrt{2}}\cos\theta}$$