Answer on Question#52704 – Math – Calculus

**Question:**

Solve the following with the help of Banoulli equation?

$$x(2+x)\frac{dy}{dx} + 2(1+x)\,y = 1 + 3x^2$$

$$y(-1) = 1$$

**Solution.**

$$x(2+x)y' + 2(1+x)y = 1 + 3x^2 \qquad | : x(2+x)$$

$$y' + \frac{2(1+x)}{x(2+x)}y = \frac{1+3x^2}{x(2+x)}$$

Firstly we will solve corresponding homogeneous equation

$$y' + \frac{2(1+x)}{x(2+x)}y = 0$$

$$y' = -\frac{2(1+x)}{x(2+x)}y$$

$$\frac{dy}{y} = -\frac{2(1+x)}{x(2+x)}dx$$

$$\frac{dy}{y} = -\frac{(2+x)+x}{x(2+x)}dx$$

$$\frac{dy}{y} = \left(-\frac{(2+x)}{x(2+x)} - \frac{x}{x(2+x)}\right)dx$$

$$\frac{dy}{y} = \left(-\frac{1}{x} - \frac{1}{2+x}\right)dx$$

$$\ln|y| = -\ln|x| - \ln|2+x| + \ln|C|$$

$$y = \frac{C}{(2+x)x}.$$

We will use constant variation method to find a partial solution $y_*$ of nonhomogeneous equation

$$y_* = \frac{\varphi(x)}{(2+x)x}$$

Substituting this into the very first equation we get

$$x(x+2)\frac{\varphi' x\,(2+x)-(2+2x)\varphi}{x^2(2+x)^2}+2(1+x)\frac{\varphi}{x(2+x)}=1+3x^2$$

$$\frac{x^2(2+x)^2}{x^2(2+x)^2}\varphi'-\frac{2+2x}{x(2+x)}\varphi+\frac{2+2x}{x(2+x)}\varphi=1+3x^2$$

So we have

$$\varphi'=1+3x^2$$

Integrating this we get

$$\varphi=x+x^3$$

So

$$y_*=\frac{x+x^3}{(2+x)x}=\frac{1+x^2}{2+x}.$$

The solution is

$$y=\frac{C}{(2+x)x}+\frac{1+x^2}{2+x}.$$

**Answer.** $y=\frac{C}{(2+x)x}+\frac{1+x^2}{2+x}$.