Answer on Question #52703– Math – Differential Calculus | Equations

Solve the following by Bernoulli equation
dy/dx + y/2x = x/y^3

$$\frac{dy}{dx} + \frac{y}{2x} = \frac{x}{y^3}$$

Solution

Let $z = y^{1-(-3)} = y^4$, then

$$\frac{dz}{dx} = 4y^3\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{4y^3}\frac{dz}{dx}$$

$$\frac{1}{4y^3}\frac{dz}{dx} + \frac{y}{2x} = \frac{x}{y^3}$$

$$\frac{dz}{dx} + \frac{4y^4}{2x} = 4x$$

$$\frac{dz}{dx} + \frac{2}{x}z = 4x$$

This is a linear first order ordinary differential equation in the dependent variable x. We write the equation in the form:

$$\frac{dz}{dx} + P(x)z = Q(x)$$

$$\frac{dz}{dx} + \frac{2}{x}z = 4x$$

So $P(x) = \frac{2}{x}, Q(x) = 4x$

The integrating factor is

$$\mu(x) = e^{\int P(x)dx}$$

that is,

$$\mu(x) = e^{\int \frac{2}{x}dx}$$

$$\int \frac{2}{x}dx = 2\ln x$$

$$\mu(x) = e^{2\ln x} = x^2$$

Consider a differential equation

$$\frac{dz}{dx} + \frac{2}{x}z = 4x$$

multiplying by x^2 gives

$$x^2 \frac{dz}{dx} + 2xz = 4x^3$$

so, $(x^2 z)' = 4x^3$

integrating both sides with respect to x gives

$x^2 z = x^4 + C,$

whence

$z = x^2 + \frac{C}{x^2},$

where C is an arbitrary real constant.

Recalling the substitution $z = y^4$, obtain that

$y^4 = x^2 + \frac{C}{x^2},$

where C is an arbitrary real constant.

The solution to a linear first order differential equation is then

$$z(x) = \frac{\int \mu(x)Q(x)dx + C}{\mu(x)}$$

$z(x) = \frac{\int 4x^3 dx + C}{x^2} = \frac{x^4 + C}{x^2} = x^2 + \frac{C}{x^2},$

where C is an arbitrary real constant.

Answer:

$$\boldsymbol{y^4(x) = x^2 + \frac{C}{x^2}}$$