Answer on Question #52685, Math, Linear Algebra

a)Consider the function f:R\{−1}→R defined by f(x)=2x+1/ x+1. i) ii) iii) iv) Check that f(x) is well defined and 1−1. Check that f(x)=2 for any x∈R. Check that g:R\{2}→R given by g(x)=x−1 Further, check that g(x)=−1 for any x∈R. 2−x is well defined and 1−1. (20) (3) (2) (4) Check that (f∘g)(x)=x for x∈R\{2} and (g∘f)(x)=x for x∈R\{−1}.(4) b)Find the direction cosines of the perpendicular from the origin to the plane r·(6i+4j+2√ 3k)+2=0.

Answer:

a) f:R\{−1}→R $f(x)=\frac{2x+1}{x+1}$

$$f(x_1)=f(x_2)\Rightarrow\frac{2x_1+1}{x_1+1}=\frac{2x_2+1}{x_2+1}\Rightarrow(2x_1+1)(x_2+1)=(2x_2+1)(x_1+1)$$

$$2x_1x_2+2x_1+x_2+1=2x_1x_2+x_1+2x_2+1\Rightarrow x_1=x_2$$

$$f(x)=\frac{2x+1}{x+1}=2\Rightarrow 2x+1=2x+2\Rightarrow 1\neq 2$$

$x\notin R$

$$f(x)=\frac{2x+1}{x+1}=y\Rightarrow 2x+1=y(x+1)\Rightarrow$$

$$(2-y)x=y-1\Rightarrow x=\frac{y-1}{2-y}$$

R\{2}→R: $g(x)=x-1$

$$g(x_1)=g(x_2)\Rightarrow x_1-1=x_2-1\Rightarrow x_1=x_2$$

$$g(x)=-x+1=-1\Rightarrow x=2$$

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but R\{2}→R

$$(g*f)(x)=g((f(x)))=g\left(\frac{2x+1}{x+1}\right)=-\frac{2x+1}{x+1}+1=\frac{-2x+1+(x+1)}{x+1}=-\frac{x}{x+1}$$

$x\neq -1$

$$(\mathrm{f}*\mathrm{g})(x)=f((\mathrm{g}(x)))=f(x-1)=\frac{2(1-x)+1}{(1-x)+1}=\frac{3-2x}{2-x}$$

$x\neq 2$

b) r·(6i+4j+2√ 3k)+2=0

The given equation can be written as

$$\vec{r}\cdot\left(6\vec{i}+4\vec{j}+2\sqrt{3}\vec{k}\right)=-2$$

$$\left|\left(6\vec{i}+4\vec{j}+2\sqrt{3}\vec{k}\right)\right|=\sqrt{6^2+4^2+\left(2\sqrt{3}\right)^2}=8$$

Then $\frac{\vec{r}\cdot\left(6\vec{i}+4\vec{j}+2\sqrt{3}\vec{k}\right)}{8}=-\frac{2}{8}\Rightarrow\vec{r}\cdot\left(\frac{3}{4}\vec{i}+\frac{1}{2}\vec{j}+\frac{\sqrt{3}}{4}\vec{k}\right)=-\frac{1}{4}$

which is the equation of the plane in the form $\vec{r}\cdot\vec{n}=d$

The direction cosines of $\vec{n}$ are $\frac{3}{4}$, $\frac{1}{2}$, $\frac{\sqrt{3}}{4}$.