Answer on Question #52676 – Math – Linear Algebra

Question

a. Consider the function f:R\{−1}→R defined by f(x)=2x+1 /x+1.

1) Check that f(x) is well defined and 1−1.

2) Check that f(x) is not equal to 2 for any x∈R.

3) Check that g:R\{2}→R given by g(x)=x−1/2-x is well defined and 1-1. Further,check that g(x) is not equal to −1 for any x∈R.

4) Check that (f∘g)(x)=x for x∈R\{2} and (g∘f)(x)=x for x∈R\{−1}.

b. Find the direction cosines of the perpendicular from the origin to the plane r·(6i+4j+2√ 3k)+2=0.

Solution

a.

1) The function $f(x) = \frac{2x+1}{x+1}$ is defined everywhere except point $x = -1$.

Suppose that $f(x_1) = f(x_2)$, that is, $\frac{2x_1+1}{x_1+1} = \frac{2x_2+1}{x_2+1}$, which is equivalent to

$$\frac{2x_1+2-1}{x_1+1} = \frac{2x_2+2-1}{x_2+1}$$

$$2 - \frac{1}{x_1+1} = 2 - \frac{1}{x_2+1}$$

Subtract 2 from both sides of equality

$$-\frac{1}{x_1+1} = -\frac{1}{x_2+1}$$

The left-hand and right-hand sides of equality are equal, numerators in both sides are equal, hence denominators in both sides should be equal, that is,

$$x_1 + 1 = x_2 + 1$$

Subtract 1 from both sides and obtain

$x_1 = x_2$.

If $f(x_1) = f(x_2)$ implies that $x_1 = x_2$, then f is one-to-one function. This statement was proved before.

Finally, $f(x) = \frac{2x+1}{x+1}$ is one-to-one function.

2)

$$f(x) = \frac{2x+1}{x+1} = \frac{2x+2-1}{x+1} = 2 - \frac{1}{x+1}$$

So, one can easily see that *f(x)* is not equal to 2 for any x∈R, because $\frac{1}{x+1}$ is never equal to 0.

3)

Function $g(x) = \frac{x-1}{2-x}$ is defined everywhere except point $x = 2$.

Suppose that $g(x_1) = g(x_2)$, that is, $\frac{x_1-1}{2-x_1} = \frac{x_2-1}{2-x_2}$, which is equivalent to

$-\frac{x_1-1}{x_1-2} = -\frac{x_2-1}{x_2-2}$

$$-\frac{x_1-2+1}{x_1-2} = -\frac{x_2-2+1}{x_2-2}$$

$$-1 - \frac{1}{x_1-2} = -1 - \frac{1}{x_2-2}$$

Add 1 to both sides of equality

$$-\frac{1}{x_1-2} = -\frac{1}{x_2-2}$$

The left-hand and right-hand sides of equality are equal, numerators in both sides are equal, hence denominators in both sides should be equal, that is,

$x_1 - 2 = x_2 - 2$

Add 2 to both sides and obtain

$x_1 = x_2$.

If $g(x_1) = g(x_2)$ implies that $x_1 = x_2$, then g is one-to-one function. This statement was proved before.

Finally, $g(x) = \frac{x-1}{2-x}$ is one-to-one function.

4)

$$(fog)(x) = f\big(g(x)\big) = \frac{2g(x)+1}{g(x)+1} = \frac{2\cdot\frac{x-1}{2-x}+1}{\frac{x-1}{2-x}+1} = \frac{\frac{2\cdot(x-1)+1\cdot(2-x)}{2-x}}{\frac{x-1+1\cdot(2-x)}{2-x}}$$

$$= \frac{\frac{2x-2+2-x}{2-x}}{\frac{x-1+2-x}{2-x}} = \frac{\frac{x}{2-x}}{\frac{1}{2-x}} = x.$$

$$(gof)(x) = g\big(f(x)\big) = \frac{f(x)-1}{2-f(x)} = \frac{\frac{2x+1}{x+1}-1}{2-\frac{2x+1}{x+1}} = \frac{\frac{2x+1-(x+1)}{x+1}}{\frac{2(x+1)-(2x+1)}{x+1}} = \frac{\frac{2x+1-x-1}{x+1}}{\frac{2x+2-2x-1}{x+1}}$$

$$= \frac{\frac{x}{x+1}}{\frac{1}{x+1}} = x.$$

b) Normal vector for this plane r·(6i+4j+2√ 3k)+2=0 is $(6,4,2\sqrt{3})$, its length is

$$\left|(6,4,2\sqrt{3})\right| = \sqrt{36+16+12} = 8.$$

The direction cosines of the perpendicular from the origin to the plane r·(6i+4j+2√ 3k)+2=0 are

$\frac{6}{8} = \frac{3}{4}, \frac{4}{8} = \frac{1}{2}$ and $\frac{2\sqrt{3}}{8} = \frac{\sqrt{3}}{4}$.

Answer: a) Yes; **b)** $\left(\frac{3}{4}, \frac{1}{2}, \frac{\sqrt{3}}{4}\right)$.