Answer on Question #52631 – Math – Abstract Algebra

Show that subgroup and factor group of nilpotent group is nilpotent?

Proof

We use the idea of a central series for a group. The following are equivalent formulations:

- A nilpotent group is one that has a central series of finite length.
- A nilpotent group is one whose lower central series terminates in the trivial subgroup after finitely many steps.
- A nilpotent group is one whose upper central series terminates in the whole group after finitely many steps.

Suppose $Z_n(G) = G$ and let $H \leq G$. For each r, $Z_r(H) \geq Z_r(G) \cap H$ and so $Z_n(H) = H$.

Suppose now that H is normal in G. For each r, $Z_r(G)H/H \geq Z_r(G/H)$ and so

$Z_n(G/H)$ is trivial for any *n* by the method of mathematical induction.