Answer on Question #52627 – Math – Vector Calculus

Calculate the divergence of the vector field $\boldsymbol{f}$:

a) $\boldsymbol{f} = (xy, yz, y^2 - x^3)$;

b) $\boldsymbol{f} = (x, y, z)$;

c) $\boldsymbol{f} = (x - 2zx^2, z - xy, z^2x^2)$.

Solution

First of all, the definition of divergence is the following:

$$div\,\boldsymbol{f} = \frac{\partial f_x}{\partial x} + \frac{\partial f_y}{\partial y} + \frac{\partial f_z}{\partial z}.$$

Now, we can calculate divergence of the vector field $\boldsymbol{f}$ in three cases:

a) $div\,\boldsymbol{f} = \frac{\partial(xy)}{\partial x} + \frac{\partial(yz)}{\partial y} + \frac{\partial(y^2-x^3)}{\partial z} = y\frac{\partial(x)}{\partial x} + z\frac{\partial(y)}{\partial y} + 0 = y + z$;

b) $div\,\boldsymbol{f} = \frac{\partial(x)}{\partial x} + \frac{\partial(y)}{\partial y} + \frac{\partial(z)}{\partial z} = 1 + 1 + 1 = 3$;

c) $div\,\boldsymbol{f} = \frac{\partial(x-2zx^2)}{\partial x} + \frac{\partial(z-xy)}{\partial y} + \frac{\partial(z^2x^2)}{\partial z} = \left(\frac{\partial x}{\partial x} - 2z\frac{\partial(x^2)}{\partial x}\right) + \left(\frac{\partial z}{\partial y} - x\frac{\partial y}{\partial y}\right) + x^2\frac{\partial(z^2)}{\partial z} =$
$= 1 - 4zx - x + 2zx^2$.

Answer:

a) $div\,\boldsymbol{f} = y + z$;

b) $div\,\boldsymbol{f} = 3$;

c) $div\,\boldsymbol{f} = 1 - 4zx - x + 2zx^2$.