Answer on Question# #52626 – Mathematics – Multivariable Calculus

Question

1) find the volume of the region bounded by Z=8-y^(2), y=8-x^(2), x=0,y=0,z=0;

2) a) find curl(z-y^(2),x+z^(3),y+x^(2)), **b)** curl(x/y,y/z,z/x)

Solution

1) The volume of the three-dimensional region E is given by the integral

$$V = \iiint_{(E)} dxdydz. \qquad (1)$$

First, we sketch the given surfaces and planes, and then we determine the limits of integration in (1). In the three-dimensional case we have the following solid:

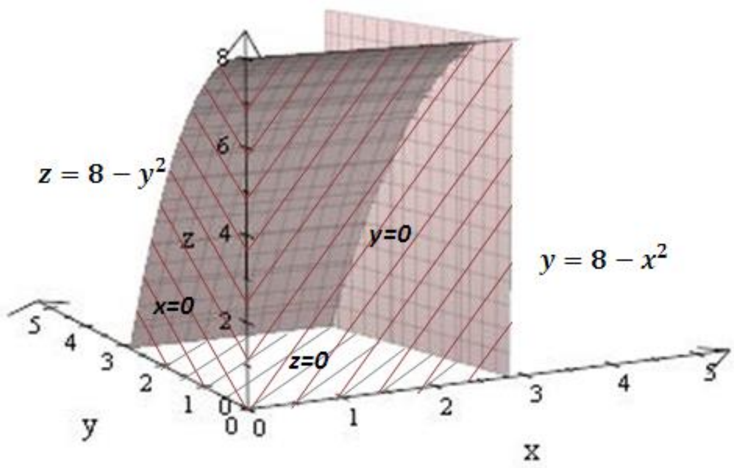

Fig.1

Let *D* is the projection of *E* onto the *xy*-plane. Then a sketch of region *D is:*

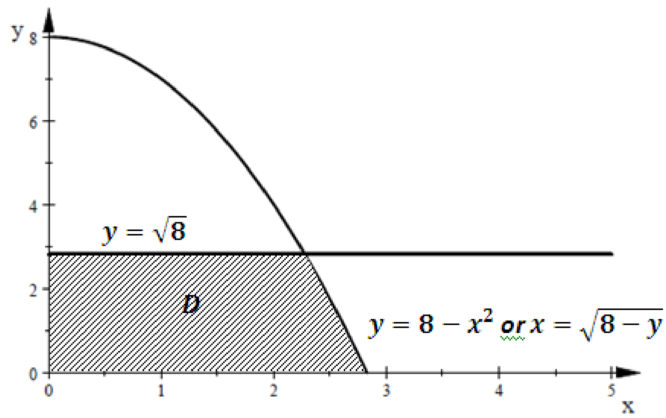

Fig.2

Now we can write the limits for each of the integration variables:

$$0 \le z \le 8 - y^2,$$
$$0 \le x \le \sqrt{8-y},$$
$$0 \le y \le \sqrt{8}.$$

Therefore, the volume is

$$V = \int_0^{\sqrt{8}} dy \int_0^{\sqrt{8-y}} dx \int_0^{8-y^2} dz = \int_0^{\sqrt{8}} dy \int_0^{\sqrt{8-y}} (8-y^2)dx = \int_0^{\sqrt{8}} dy(8-y^2)x\,|_0^{\sqrt{8-y}} = \int_0^{\sqrt{8}} (8-y^2)\sqrt{8-y}\,dy$$
$$= \int_0^{\sqrt{8}} \left(8\sqrt{8-y} - y^2\sqrt{8-y}\right) dy = -8\int_0^{\sqrt{8}} \sqrt{8-y}\,d(8-y) - \int_0^{\sqrt{8}} y^2\sqrt{8-y}\,dy$$
$$= -8 \cdot \frac{2}{3}(8-y)^{\frac{3}{2}}|_0^{\sqrt{8}} - \int_0^{\sqrt{8}} y^2\sqrt{8-y}\,dy$$
$$= -8 \cdot \frac{2}{3}\left(\left(8-\sqrt{8}\right)^{\frac{3}{2}} - (8)^{\frac{3}{2}}\right) - \left(-\frac{2}{105}(8-y)^{\frac{3}{2}}(15y^2+96y+512)|_0^{\sqrt{8}}\right)$$
$$= -\frac{16}{3}\left(\left(8-\sqrt{8}\right)^{\frac{3}{2}} - (8)^{\frac{3}{2}}\right) + \frac{2}{105}\left(\left(8-\sqrt{8}\right)^{\frac{3}{2}} - (8)^{\frac{3}{2}}\right)512$$
$$+\frac{2}{105}\left(\left(8-\sqrt{8}\right)^{\frac{3}{2}}(15\cdot 8 + 96\cdot\sqrt{8})\right)$$
$$= \frac{2}{105}\left(\left(8-\sqrt{8}\right)^{\frac{3}{2}} - (8)^{\frac{3}{2}}\right)\cdot 232 + \frac{2}{105}\left(\left(8-\sqrt{8}\right)^{\frac{3}{2}}(15\cdot 8 + 96\cdot\sqrt{8})\right)$$
$$\cong 39.7\ cubic\ units.$$

2) Let $\vec{F} = \{P(x,y,z), Q(x,y,z), R(x,y,z)\}$ is a vector field. Then the curl of this vector field curl $\vec{F}$ (or $\vec{\nabla} \times \vec{F}$), is defined as

$$\vec{A} = \mathrm{curl}\,\vec{F} = \vec{\nabla} \times \vec{F} = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ P & Q & R \end{vmatrix} = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\vec{\imath} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\vec{\jmath} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\vec{k}. \qquad (2)$$

As we see, the curl $\vec{F}$ is a vector.

a) curl($z - y^2, x + z^3, y + x^2$).

$$\vec{a} = \operatorname{curl}\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ z - y^2 & x + z^3 & y + x^2 \end{vmatrix}$$

$$= \left(\frac{\partial}{\partial y}(y + x^2) - \frac{\partial}{\partial z}(x + z^3)\right)\vec{i} + \left(\frac{\partial}{\partial z}(z - y^2) - \frac{\partial}{\partial x}(y + x^2)\right)\vec{j}$$

$$+ \left(\frac{\partial}{\partial x}(x + z^3) - \frac{\partial}{\partial y}(z - y^2)\right)\vec{k} = (1 - 3z^2)\vec{i} + (1 - 2x)\vec{j} + (1 + 2y)\vec{k}$$

$$= \{(1 - 3z^2), (1 - 2x), (1 + 2y)\}.$$

b) curl$(\frac{x}{y}, \frac{y}{z}, \frac{z}{x})$.

$$\vec{b} = \operatorname{curl}\vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ \dfrac{x}{y} & \dfrac{y}{z} & \dfrac{z}{x} \end{vmatrix} = \left(\frac{\partial}{\partial y}\left(\frac{z}{x}\right) - \frac{\partial}{\partial z}\left(\frac{y}{z}\right)\right)\vec{i} + \left(\frac{\partial}{\partial z}\left(\frac{x}{y}\right) - \frac{\partial}{\partial x}\left(\frac{z}{x}\right)\right)\vec{j} + \left(\frac{\partial}{\partial x}\left(\frac{y}{z}\right) - \frac{\partial}{\partial y}\left(\frac{x}{y}\right)\right)\vec{k}$$

$$= \left(\frac{y}{z^2}\right)\vec{i} + \left(\frac{z}{x^2}\right)\vec{j} + \left(\frac{x}{y^2}\right)\vec{k} = \left\{\frac{y}{z^2}, \frac{z}{x^2}, \frac{x}{y^2}\right\}.$$

Answer: **1)** $V \cong 39.7\ cubic\ units.$

2) a) $\vec{a} = \operatorname{curl}\vec{F} = \{(1 - 3z^2), (1 - 2x), (1 + 2y)\}$; **b)** $\vec{b} = \operatorname{curl}\vec{F} = \left\{\frac{y}{z^2}, \frac{z}{x^2}, \frac{x}{y^2}\right\}$.