Answer on Question #52574 – Math – Combinatorics | Number Theory

Q. For n belongs to Z or n>0 c belongs to Z. Prove that $10^n + 3 \cdot 4^{n+2} + 5$ is divisible by 9.

Solution

The principle of mathematical induction (P.M.I.) will be applied.

Let $f(n) = 10^n + 3 \cdot 4^{n+2} + 5, n \in \mathbb{Z}$

Let $n = 1$

$f(1) = 10^1 + 3 \cdot 4^{1+2} + 5 = 10 + 192 + 5 = 207 = 9 \cdot 23$

$f(1)$ is divisible by 9 i.e., the result is true for $n = 1$

Let the result be true for $n = k$

Assume that $f(k) = 10^k + 3 \cdot 4^{k+2} + 5$ is divisible by 9

Let $10^k + 3 \cdot 4^{k+2} + 5 = 9 \cdot M$, $M \in \mathbb{Z}$.

Let $n = k + 1$

$$10^{k+1} + 3 \cdot 4^{(k+1)+2} + 5 = 10 \cdot 10^k + 3 \cdot 4^{k+3} + 5 = 10\left(9M - 3 \cdot 4^{k+2} - 5\right) + 3 \cdot 4^{k+3} + 5 =$$
$$= 90M + 3 \cdot 4^{k+3} - 10 \cdot 3 \cdot 4^{k+2} + 50 - 5 = 90M + 3 \cdot 4^{k+2}(4 - 10) + 45 = 90M - 18 \cdot 4^{k+2} + 45 =$$
$$= 9\left(10M - 2 \cdot 4^{k+2} + 5\right)$$

By P.M.I., $10^n + 3 \cdot 4^{n+2} + 5$ is divisible by 9.