Answer on Question #52554 – Math – Calculus

Question

Find the Fourier transform of the following function: $f(x) = \begin{cases} 1, |x| < a \\ 0, |x| \geq a \end{cases}$. Hence evaluate $\int_{-\infty}^{\infty} \frac{\sin ax}{x} dx$.

Solution

Since function $f(x)$ is even then we have:

$$\hat{f}(y) = \int_{-\infty}^{\infty} f(x) e^{-ixy} dx = \int_{-a}^{a} e^{-ixy} dx = 2 \int_{0}^{a} \cos xy \, dx = 2 \frac{\sin ay}{y}, y \in \mathbb{R}.$$

Now using the formula of inverse transform we obtain:

$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \hat{f}(y) e^{ixy} dy = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin ay}{y} e^{ixy} dy$. Letting $x = 0$ we conclude that

$$f(0) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin ay}{y} dy \Rightarrow \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin ay}{y} dy = 1 \Rightarrow \int_{-\infty}^{\infty} \frac{\sin ay}{y} dy = \pi.$$

Answer: $\hat{f}(y) = 2 \frac{\sin ay}{y}$; $\int_{-\infty}^{\infty} \frac{\sin ax}{x} dx = \pi$.