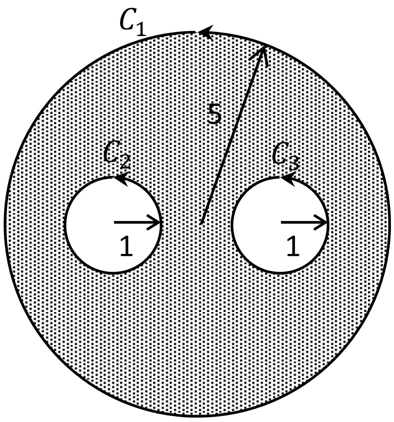
Answer on Question #52366 - Math – Multivariable Calculus

Evaluate the circulation of F around C_1, $\oint_{C_1} F \cdot ds$, if

$$\oint_{C_2} F \cdot ds = 3\pi \qquad \oint_{C_3} F \cdot ds = 4\pi$$

and $\mathbf{curl}_z(F) \equiv 9$ on the shaded region.



Solution:

According to the Stokes' theorem

$$\oint_{C_1 - C_2 - C_3} F \cdot ds = \iint_{\Sigma} |\mathbf{curl}_z(F)| d\Sigma,$$

where Σ – is the shaded region. Since $|\mathbf{curl}_z(F)| = const = 9$, we obtain

$$\iint_{\Sigma} |\mathbf{curl}_z(F)| d\Sigma = |\mathbf{curl}_z(F)| \cdot A,$$

where A is the area of shaded region, which is

$$A = \pi(5)^2 - \pi(1)^2 - \pi(1)^2 = 23\pi$$

Therefore,

$$\oint_{C_1 - C_1 - C_1} F \cdot ds = \oint_{C_1} F \cdot ds - \oint_{C_2} F \cdot ds - \oint_{C_3} F \cdot ds = \iint_{\Sigma} |\mathbf{curl}_z(F)| d\Sigma$$

$$\oint_{C_1} F \cdot ds = \oint_{C_2} F \cdot ds + \oint_{C_3} F \cdot ds + |\mathbf{curl}_z(F)| \cdot A = 3\pi + 4\pi + 23\pi = 30\pi$$

Answer: 30π.