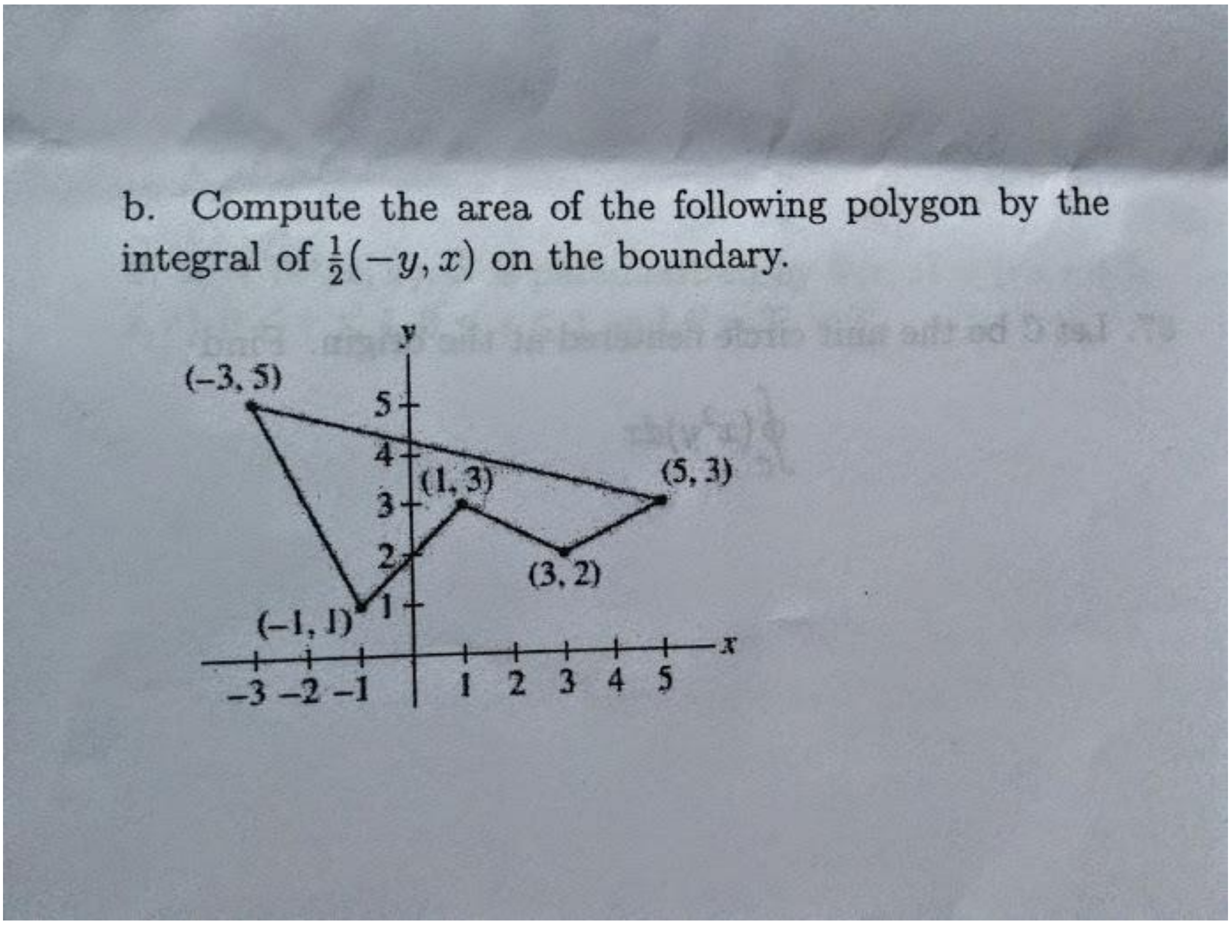
Answer on Question #52363 - Math - Multivariable Calculus


b. Compute the area of the following polygon by the integral of $\frac{1}{2}(-y, x)$ on the boundary.

<u>Solution:</u>

Area:

$$A = \frac{1}{2}\oint -ydx + xdy =$$

$$= \frac{1}{2}\left[\oint_{(-3,5)}^{(-1,1)} (-ydx + xdy) + \oint_{(-1,1)}^{(1,3)} (-ydx + xdy) + \oint_{(1,3)}^{(3,2)} (-ydx + xdy) + \right.$$

$$\left. + \oint_{(3,2)}^{(5,3)} (-ydx + xdy) + \oint_{(5,3)}^{(-3,5)} (-ydx + xdy)\right]$$

Equation of line through points $(-3, 5), (-1, 1)$ is $y = -2x - 1$.

Besides,

$$dy = -2dx$$

$$A_1 = \frac{1}{2}\oint_{(-3,5)}^{(-1,1)} (-ydx + xdy) = \frac{1}{2}\int_{-3}^{-1} (2x + 1 - 2x)dx = \frac{1}{2}\int_{-3}^{-1} dx = \frac{1}{2}x\Big|_{-3}^{-1}$$
$$= \frac{1}{2}(-1 - (-3)) = \frac{3-1}{2} = 1$$

Equation of line through points $(-1, 1), (1, 3)$ is $y = x + 2$.

Besides, $dy = dx$.

$$A_2 = \frac{1}{2}\oint_{(-1,1)}^{(1,3)} (-ydx + xdy) = \frac{1}{2}\int_{-1}^{1}(-x - 2 + x)dx = -\int_{-1}^{1} dx = -x|_{-1}^{1} = x|_{1}^{-1} = -1 - 1$$
$$= -2$$

Equation of line through points $(1, 3), (3, 2)$ is $y = -\frac{1}{2}x + \frac{7}{2}$.

Besides, $dy = -\frac{1}{2}dx$.

$$A_3 = \frac{1}{2}\oint_{(1,3)}^{(3,2)} (-ydx + xdy) = \frac{1}{2}\int_{1}^{3}\left(\frac{1}{2}x - \frac{7}{2} - \frac{1}{2}x\right)dx = -\frac{7}{4}\int_{1}^{3} dx = -\frac{7}{4}x\Big|_{1}^{3} = -\frac{7}{4}(3-1)$$
$$= -\frac{7}{2}$$

Equation of line through points $(3, 2), (5, 3)$ is $y = \frac{1}{2}x + \frac{1}{2}$.

Besides, $dy = \frac{1}{2}dx$.

$$A_4 = \frac{1}{2}\oint_{(3,2)}^{(5,3)} (-ydx + xdy) = \frac{1}{2}\int_{3}^{5}\left(-\frac{1}{2}x - \frac{1}{2} + \frac{1}{2}x\right)dx = -\frac{1}{4}\int_{3}^{5} dx = -\frac{x}{4}\Big|_{3}^{5}$$
$$= -\frac{1}{4}(5-3) = -\frac{1}{2}$$

Equation of line through points $(5, 3), (-3, 5)$ is $y = -\frac{1}{4}x + \frac{17}{4}$.

Besides, $dy = -\frac{1}{4}dx$.

$$A_5 = \frac{1}{2}\oint_{(5,3)}^{(-3,5)} (-ydx + xdy) = \frac{1}{2}\int_{5}^{-3}\left(\frac{1}{4}x - \frac{17}{4} - \frac{1}{4}x\right)dx = -\frac{17x}{8}\Big|_{5}^{-3} = -\frac{17}{8}(-3-5)$$
$$= 17$$

Therefore,

$$A = A_1 + A_2 + A_3 + A_4 + A_5 = 1 - 2 - \frac{7}{2} - \frac{1}{2} + 17 = 12.$$

Answer: 12.