Answer on Question #52362 – Math – Calculus

Let $f(x) = arctanx$. Find $f'(x)$.

Solution

Let $f(x) = arctan(x) \Rightarrow \tan\big(f(x)\big) = x$.

Where x is any real number and $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$. Suppose we know that $\frac{dtan(y)}{dy} = \frac{1}{cos^2 y} = sec^2 y$, but do not know $\frac{d(arctany)}{dy}$.

Differentiate both sides of $\tan\big(f(x)\big) = x$ with respect to x:

$$\frac{d}{dx}\Big(\tan\big(f(x)\big)\Big) = \frac{d}{dx}(x),$$

$$sec^2\big(f(x)\big) * \frac{d\big(f(x)\big)}{dx} = 1,$$

$$\frac{d\big(f(x)\big)}{dx} = \frac{1}{sec^2\big(f(x)\big)}.$$

However,

$$sec^2\big(f(x)\big) = 1 + tan^2\big(f(x)\big) = 1 + x^2.$$

Thus, if $f(x) = arctan(x)$, then $f'(x) = \frac{1}{1+x^2}$.

Answer: $f'(x) = \frac{1}{1+x^2}$.