Answer on Question #52361 – Math – Vector Calculus

Question

For the following vectors
a = (2,3,5), b = (4,6,8), c = (-1,-5,9)
Calculate
a·(b × c)

Solution

Method 1

The cross (or vector) product is

$$b \times c = \begin{vmatrix} i & j & k \\ 4 & 6 & 8 \\ -1 & -5 & 9 \end{vmatrix} = \begin{vmatrix} 6 & 8 \\ -5 & 9 \end{vmatrix} i - \begin{vmatrix} 4 & 8 \\ -1 & 9 \end{vmatrix} j + \begin{vmatrix} 4 & 6 \\ -1 & -5 \end{vmatrix} k = (6 \cdot 9 - (-5) \cdot 8)i -$$
$$-(4 \cdot 9 - (-1) \cdot 8)j + (4 \cdot (-5) - (-1) \cdot 6)k = 94i - 44j - 14k = (94, -44, -14)$$

The dot (or scalar) product of vectors a and $b \times c$ is

$$a \cdot (b \times c) = (2,3,5) \cdot (94, -44, -14) = 2 \cdot 94 - 3 \cdot 44 - 5 \cdot 14 = -14$$

Method 2

The scalar triple product is

$$a \cdot (b \times c) = \begin{vmatrix} a_x & a_y & a_z \\ b_x & b_y & b_z \\ c_x & c_y & c_z \end{vmatrix} = \begin{vmatrix} 2 & 3 & 5 \\ 4 & 6 & 8 \\ -1 & -5 & 9 \end{vmatrix} = 2 \begin{vmatrix} 6 & 8 \\ -5 & 9 \end{vmatrix} - 3 \begin{vmatrix} 4 & 8 \\ -1 & 9 \end{vmatrix} + 5 \begin{vmatrix} 4 & 6 \\ -1 & -5 \end{vmatrix} ==$$
$$2(6 \cdot 9 - (-5) \cdot 8) - 3(4 \cdot 9 - (-1) \cdot 8) + 5 \cdot (4 \cdot (-5) - (-1) \cdot 6) == 2 \cdot (54 + 40) -$$
$$3(36 + 8) + 5(-20 + 6) = 2 \cdot 94 - 3 \cdot 44 - 5 \cdot 14 = -14.$$

Answer: -14.