# Answer on Question #52359 – Math – Vector Calculus

For the following vectors $\vec{a} = (3,5,7)$ and $\vec{b} = (4,6,8)$ calculate the following:

a) $\vec{a} \times \vec{b}$

b) $\vec{b} \times \vec{a}$ .

***Solution***

The cross product or vector product between $\vec{a}$ and $\vec{b}$ is written as $\vec{a} \times \vec{b}$ . The result of a cross product is a new vector $\vec{c} = \vec{a} \times \vec{b}$ . Magnitude of $\vec{c}$ is defined as $|\vec{c}| = |\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$, where $\theta$ is the angle between $\vec{a}$ and $\vec{b}$ when both of vectors are drawn 'tail-o-tail'. The vector $\vec{c}$ is perpendicular to the plane formed by $\vec{a}$ and $\vec{b}$ .

The cross product is anticommutative: $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$ .

Let's evaluate the cross product using $\vec{a}$ and $\vec{b}$ in component form:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \vec{i}(a_y b_z - a_z b_y) - \vec{j}(a_x b_z - a_z b_x) + \vec{k}(a_x b_y - a_y b_x) .$$

**a)**
$$\vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 5 & 7 \\ 4 & 6 & 8 \end{vmatrix} = \vec{i}(5 \cdot 8 - 7 \cdot 6) - \vec{j}(3 \cdot 8 - 4 \cdot 7) + \vec{k}(3 \cdot 6 - 5 \cdot 4) = \vec{i}(-2) - \vec{j}(-4) + \vec{k}(-2) =$$
$$= -2\vec{i} + 4\vec{j} - 2\vec{k}.$$

**b)** **First method** (straightforward computation)
$$\vec{b} \times \vec{a} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & 6 & 8 \\ 3 & 5 & 7 \end{vmatrix} = \vec{i}(7 \cdot 6 - 5 \cdot 8) - \vec{j}(4 \cdot 7 - 3 \cdot 8) + \vec{k}(5 \cdot 4 - 3 \cdot 6) = \vec{i}(2) - \vec{j}(4) + \vec{k}(2) =$$
$$= 2\vec{i} - 4\vec{j} + 2\vec{k}.$$

**Second method** (using properties of cross product)

Apply result from a) $\vec{a} \times \vec{b} = -2\vec{i} + 4\vec{j} - 2\vec{k}$ and the next property of cross product:

$$\vec{b} \times \vec{a} = -\vec{a} \times \vec{b} = -\left(-2\vec{i} + 4\vec{j} - 2\vec{k}\right) = 2\vec{i} - 4\vec{j} + 2\vec{k}.$$

***Answer:***

***a)*** $\vec{a} \times \vec{b} = -2\vec{i} + 4\vec{j} - 2\vec{k}$;

***b)*** $\vec{b} \times \vec{a} = 2\vec{i} - 4\vec{j} + 2\vec{k}$.