Answer on Question #52356 – Mathematics – Vector Calculus

Question

For the following vectors: a = (1,-5,9), b = (-4,12,-6), c = (-3,5,-7). Calculate the following dot products:

a) a·b;

b) a·c;

c) b·c.

Solution

Let us write the formula for the dot (or scalar) product in terms of vector components. For the two three-dimensional vectors

$$\vec{a} = (a_1, a_2, a_3) = a_1\vec{i} + a_2\vec{j} + a_3\vec{k} \tag{1}$$

and

$$\vec{b} = (b_1, b_2, b_3) = b_1\vec{i} + b_2\vec{j} + b_3\vec{k}, \tag{2}$$

where $\vec{i}, \vec{j}, \vec{k}$ – the standard unit vectors of orthogonal coordinate system, the dot product is defined as

$$(\vec{a} \cdot \vec{b}) = a_1 b_1 + a_2 b_2 + a_3 b_3. \tag{3}$$

Hence, using (1)-(3) for the given vectors we get:

a)

$$(\vec{a} \cdot \vec{b}) = 1 \cdot (-4) + (-5) \cdot 12 + 9 \cdot (-6) = -4 - 60 - 54 = -118,$$

b)

$$(\vec{a} \cdot \vec{c}) = 1 \cdot (-3) + (-5) \cdot 5 + 9 \cdot (-7) = -3 - 25 - 63 = -91,$$

c)

$$(\vec{b} \cdot \vec{c}) = (-4) \cdot (-3) + 12 \cdot 5 + (-6) \cdot (-7) = 12 + 60 + 42 = 114.$$

Answer: **a)** $(\vec{a} \cdot \vec{b}) = -118$; **b)** $(\vec{a} \cdot \vec{c}) = -91$; **c)** $(\vec{b} \cdot \vec{c}) = 114$.