

Answer on Question #52353 – Math – Calculus

Differentiate the following functions:

a) $y(x) = x \ln x$;

b) $g(x) = \frac{x^3 - 5x}{\cot x}$.

Solution.

a) Using the product rule $(f \cdot g)' = f' \cdot g + f \cdot g'$ and $x' = 1$, $(\ln x)' = \frac{1}{x}$, we have:

$$(x \ln x)' = x' \cdot \ln x + x \cdot (\ln x)' = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1.$$

b) Using the quotient rule $\left(\frac{f}{g}\right)' = \frac{f' \cdot g - g' \cdot f}{g^2}$ and $(x^n)' = nx^{n-1}$, $(r - s)' = r' - s'$,

$(\cot x)' = -\frac{1}{\sin^2 x}$, we have:

$$\begin{aligned} \left(\frac{x^3 - 5x}{\cot x}\right)' &= \frac{(x^3 - 5x)' \cdot \cot x - (x^3 - 5x) \cdot (\cot x)'}{\cot^2 x} = \\ &= \frac{(3x^2 - 5) \cdot \cot x - (x^3 - 5x) \cdot \left(-\frac{1}{\sin^2 x}\right)}{\cot^2 x} = \frac{3x^2 - 5}{\cot x} + \frac{x^3 - 5x}{\cos^2 x} = \\ &= (3x^2 - 5) \cdot \tan x + (x^3 - 5x) \cdot \sec^2 x \end{aligned}$$

Answer: a) $\ln(x) + 1$; b) $(3x^2 - 5) \tan(x) + (x^3 - 5x) \sec^2(x)$.