Answer on Question #52350 – Math – Real Analysis

For the functions

$f(x) = x^2 + 1$

$g(x) = \dfrac{e^x}{(x-1)}$

Calculate $(f \circ g)(x)$ and $(g \circ f)(x)$.

What are the domains of $(f \circ g)(x)$ and $(g \circ f)(x)$?

Solution:

$$(f \circ g)(x) = f\big(g(x)\big) = \left(\frac{e^x}{(x-1)}\right)^2 + 1$$

$(f \circ g)(x)$ is not defined at $x = 1$, as this value would result in division by zero.
Hence the domain of $(f \circ g)(x)$ is all real numbers except 1.

$$(g \circ f)(x) = g\big(f(x)\big) = \frac{e^{(x^2+1)}}{(x^2+1-1)} = \frac{e^{(x^2+1)}}{x^2}$$

$(g \circ f)(x)$ is not defined at $x = 0$, as this value would result in division by zero.
Hence the domain of $(f \circ g)(x)$ is all real numbers except 0.