

## Answer on Question #52314 – Math – Calculus

What is the integral of  $(1 + \cos 4x)^2 dx$  ?

### Solution

#### Method 1

$$\int (1 + \cos 4x)^2 dx = \int (1 + \cos^2 4x + 2\cos 4x) dx = \int (1 + 2\cos 4x) dx + \int \cos^2 4x dx =$$

$$= x + \frac{2\sin 4x}{4} + \int \cos^2 4x dx =$$

using trigonometric formula  $\cos^2 x = \frac{1 + \cos 2x}{2}$  we get

$$= x + \frac{2\sin 4x}{4} + \int \frac{1 + \cos 8x}{2} dx = x + \frac{\sin 4x}{2} + \frac{x}{2} + \frac{\sin 8x}{16} + C = \frac{3x}{2} + \frac{\sin 8x}{16} + \frac{\sin 4x}{2} + C,$$

where  $C$  is an arbitrary real constant.

$$\text{Answer: } \frac{3x}{2} + \frac{\sin 8x}{16} + \frac{\sin 4x}{2} + C$$

#### Method 2

It follows that  $1 + \cos 2t = 2\cos^2 t$  from the double - angle formula

$$\cos(2t) = \cos^2 t - \sin^2 t = 2\cos^2 t - 1 = 1 - 2\sin^2 t .$$

Compute

$$\int (1 + \cos 4x)^2 dx = \int (2\cos^2(2x))^2 dx = \int 4\cos^4(2x) dx =$$

=using power-reduction formula  $\cos^4 t = \frac{3 + 4\cos 2t + \cos 4t}{8}$  we get=

$$= \int 4 \frac{3 + 4\cos 4x + \cos 8x}{8} dx = \int \frac{3}{2} dx + 2 \int \cos 4x dx + \frac{1}{2} \int \cos 8x dx = \frac{3}{2}x + \frac{2}{4}\sin 4x + \frac{1}{16}\sin 8x + C =$$

$$= \frac{3x}{2} + \frac{\sin 8x}{16} + \frac{\sin 4x}{2} + C,$$

where  $C$  is an arbitrary real constant.