

Answer on Question #52241 – Math – Vector Calculus

What are vectors that are not parallel to the same line, called?

scalar

collinear vectors

non-collinear vectors

vectors

Answer: non-collinear vectors.

7 Find the vector product $\vec{a} \times \vec{b}$. If $\vec{a} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ and $\vec{b} = 5\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

$11\mathbf{i} + 18\mathbf{j} - 19\mathbf{k}$

$2\mathbf{j} + 3\mathbf{k}$

$5\mathbf{i} - 6\mathbf{j} + 7\mathbf{k}$

$4\mathbf{i} - 6\mathbf{j} + 11\mathbf{k}$

Solution

$$\vec{a} \times \vec{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 3 & 4 \\ 5 & -2 & 1 \end{vmatrix} = \mathbf{i}(3 \cdot 1 - 4(-2)) + \mathbf{j}(4 \cdot 5 - 1(2)) + \mathbf{k}(2 \cdot (-2) - 3(5)) = 11\mathbf{i} + 18\mathbf{j} - 19\mathbf{k}.$$

Answer: $11\mathbf{i} + 18\mathbf{j} - 19\mathbf{k}$.

8 Find the scalar product $\vec{a} \cdot \vec{b}$. If $\vec{a} = 2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$ and $\vec{b} = 5\mathbf{i} - 2\mathbf{j} + \mathbf{k}$

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8

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Solution

$$\vec{a} \cdot \vec{b} = (2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})(5\mathbf{i} - 2\mathbf{j} + \mathbf{k}) = 2 \cdot 5 + 3 \cdot (-2) + 4 \cdot 1 = 8.$$

Answer: 8.

9 The centroid of a triangle of the triangle OAB is denoted by G. If o is the origin and $\text{line}(\text{OA}) = 4\mathbf{i} + 3\mathbf{j}$, $\text{line}(\text{OB}) = 6\mathbf{i} - \mathbf{j}$, find $\text{line}(\text{OG})$ in terms of the unit vectors \mathbf{i} and \mathbf{j}

$10\mathbf{i} - 3\mathbf{j}$

$$\frac{1}{2}(10i-2j)$$

$$10i + 2j$$

$$\frac{1}{3}(10i+2j)$$

Solution

Vector $\overline{OB} = \overline{OA} + \overline{AB}$, hence $\overline{AB} = \overline{OB} - \overline{OA}$, $\overline{AM} = \frac{1}{2} \overline{AB}$.

Vector $\overline{OM} = \overline{OA} + \overline{AM} = \overline{OA} + \frac{1}{2} \overline{AB} = \overline{OA} + \frac{1}{2}(\overline{OB} - \overline{OA}) = \frac{1}{2}(\overline{OB} + \overline{OA})$.

Let OM be the median of the triangle OAB. By properties of centroid, $OG = \frac{2}{3}OM$.

Thus,

$$\overline{OG} = \frac{2}{3}\overline{OM} = \frac{2}{3} \cdot \frac{1}{2}(\overline{OB} + \overline{OA}) = \frac{1}{3}(4i + 3j + 6i - j) = \frac{1}{3}(10i + 2j).$$

Answer: $\frac{1}{3}(10i+2j)$.

10 Given that $a = 5i + 2j - k$ and $b = i - 3j + k$. Find $(a + b) \times (a - b)$.

$$2i - 12j - 34k$$

$$2i + 12j + 34k$$

$$2i - 3j + 12j$$

$$2i + 2k$$

Solution

$$\overline{(a + b)} = 5i + 2j - k + i - 3j + k = 6i - j.$$

$$\overline{(a - b)} = 5i + 2j - k - i + 3j - k = 4i + 5j - 2k.$$

$$\overline{(a + b)} \times \overline{(a - b)} = \begin{vmatrix} i & j & k \\ 6 & -1 & 0 \\ 4 & 5 & -2 \end{vmatrix} = 2i + 12j + 34k.$$

Answer: $2i+12j+34k$.