Answer on Question #52239 – Math – Vector Calculus

6. If U = I + 3j - 2k and V = 4i - 2j - 4k are vectors, find (2U + V) . (U - 2V)

4

3

5

6

7. If r4=r1+r2+r3

Which of the vectors are linearly dependence on each .

r1

r3

r2

r4

8 A vector quantity has both magnitude and ----.

direction

time

magnitude

scalar

9 What does the symbol i,j,k denote

distance

velocity

unit vector

angle

10 Evaluate the vectors (2i - 3j).[(I +j - k) x (3i - k)]

7

4

6

2

Solution

6.

Method 1

$$(2\boldsymbol{U}+\boldsymbol{V})\cdot(\boldsymbol{U}-2\boldsymbol{V})=$$

$$=[2(\boldsymbol{i}+3\boldsymbol{j}-2\boldsymbol{k})+(4\boldsymbol{i}-2\boldsymbol{j}-4\boldsymbol{k})]\cdot[(\boldsymbol{i}+3\boldsymbol{j}-2\boldsymbol{k})-2(4\boldsymbol{i}-2\boldsymbol{j}-4\boldsymbol{k})]=$$

$$=(6\boldsymbol{i}+4\boldsymbol{j}-8\boldsymbol{k})\cdot(-7\boldsymbol{i}+7\boldsymbol{j}+6\boldsymbol{k})=-42+28-48=-62.$$

Method 2

$$(2\boldsymbol{U}+\boldsymbol{V})\cdot(\boldsymbol{U}-2\boldsymbol{V})=2\boldsymbol{U}\cdot\boldsymbol{U}-2\boldsymbol{U}\cdot2\boldsymbol{V}+\boldsymbol{V}\cdot\boldsymbol{U}-\boldsymbol{V}\cdot2\boldsymbol{V}=$$

$$=2\boldsymbol{U}^2-3\boldsymbol{U}\boldsymbol{V}-2\boldsymbol{V}^2=2(1^2+3^2+(-2)^2)-3\big(1\cdot4+3\cdot(-2)+(-2)\cdot(-4)\big)-2(4^2+(-2)^2+(-4)^2)=2\cdot14-3\cdot6-2\cdot36=28-18-72=$$
$$=-62.$$

Answer: - 62.

7. $\boldsymbol{r_4}$

8. **direction**

9. **unit vector**

10.

Method 1.

If $\boldsymbol{a} = (\boldsymbol{a_1}; \boldsymbol{a_2}; \boldsymbol{a_3}), \boldsymbol{b} = (\boldsymbol{b_1}; \boldsymbol{b_2}; \boldsymbol{b_3}), \boldsymbol{c} = (\boldsymbol{c_1}; \boldsymbol{c_2}; \boldsymbol{c_3})$**, then** $\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c})$ **is the scalar triple product of** $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}$ **and it is calculated by the following formula:**

$$\boldsymbol{a} \cdot (\boldsymbol{b} \times \boldsymbol{c}) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

(in other words, it is the determinant).

Thus, $(2\boldsymbol{i} - 3\boldsymbol{j}) \cdot [(\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}) \times (3\boldsymbol{i} - \boldsymbol{k})] = \begin{vmatrix} 2 & -3 & 0 \\ 1 & 1 & -1 \\ 3 & 0 & -1 \end{vmatrix} =$

$$= (\textit{subtract the third row from the second row}) = \begin{vmatrix} 2 & -3 & 0 \\ -2 & 1 & 0 \\ 3 & 0 & -1 \end{vmatrix} =$$

$$= (\textit{expand the determinant over the third column}) =$$

$$= -1 \begin{vmatrix} 2 & -3 \\ -2 & 1 \end{vmatrix} =$$

$$= -\big(2 \cdot 1 - (-2) \cdot (-3)\big) = -(2 - 6) = 4$$

To express $(\boldsymbol{i} + \boldsymbol{j} - \boldsymbol{k}) \times (3\boldsymbol{i} - \boldsymbol{k})$**, two methods will be applied.**

Method 2

$$(\boldsymbol{i}+\boldsymbol{j}-\boldsymbol{k})\times(3\boldsymbol{i}-\boldsymbol{k})=\begin{vmatrix}\boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k}\\ 1 & 1 & -1\\ 3 & 0 & -1\end{vmatrix}=\begin{vmatrix}1 & -1\\ 0 & -1\end{vmatrix}\boldsymbol{i}-\begin{vmatrix}1 & -1\\ 3 & -1\end{vmatrix}\boldsymbol{j}+$$

$$+\begin{vmatrix}1 & 1\\ 3 & 0\end{vmatrix}\boldsymbol{k}=\big(1\cdot(-1)-0\cdot(-1)\big)\boldsymbol{i}-\big(1\cdot(-1)-3\cdot(-1)\big)\boldsymbol{j}+$$

$$+(1\cdot 0-3\cdot 1)\boldsymbol{k}=-\boldsymbol{i}-2\boldsymbol{j}-3\boldsymbol{k}$$

Method 3

By properties of vector (or cross) product, $\boldsymbol{i}\times\boldsymbol{i}=\boldsymbol{j}\times\boldsymbol{j}=\boldsymbol{k}\times\boldsymbol{k}=0$**,** $\boldsymbol{i}\times\boldsymbol{j}=\boldsymbol{k}$**,** $\boldsymbol{j}\times\boldsymbol{k}=\boldsymbol{i}$**,** $\boldsymbol{k}\times\boldsymbol{i}=\boldsymbol{j}$**,**

$\boldsymbol{a}\times\boldsymbol{b}=-\boldsymbol{b}\times\boldsymbol{a}$**,**

hence, $\boldsymbol{i}\times\boldsymbol{k}=-\boldsymbol{j}$**,** $\boldsymbol{j}\times\boldsymbol{i}=-\boldsymbol{k}$**.**

Besides,

$\boldsymbol{a}\times(\boldsymbol{b}+\boldsymbol{c})=\boldsymbol{a}\times\boldsymbol{b}+\boldsymbol{a}\times\boldsymbol{c}$**,**

$(\lambda\boldsymbol{a})\times\boldsymbol{b}=\boldsymbol{a}\times(\lambda\boldsymbol{b})=\lambda(\boldsymbol{a}\times\boldsymbol{b})$

These formulae allow to simplify

$$(\boldsymbol{i}+\boldsymbol{j}-\boldsymbol{k})\times(3\boldsymbol{i}-\boldsymbol{k})=(\boldsymbol{i}\times 3\boldsymbol{i})+\big(\boldsymbol{i}\times(-\boldsymbol{k})\big)+(\boldsymbol{j}\times 3\boldsymbol{i})+\big(\boldsymbol{j}\times(-\boldsymbol{k})\big)+$$
$$+(-\boldsymbol{k}\times 3\boldsymbol{i})+\big(-\boldsymbol{k}\times(-\boldsymbol{k})\big)=3(\boldsymbol{i}\times\boldsymbol{i})-(\boldsymbol{i}\times\boldsymbol{k})+3(\boldsymbol{j}\times\boldsymbol{i})-(\boldsymbol{j}\times\boldsymbol{k})-$$
$$-3(\boldsymbol{k}\times\boldsymbol{i})+(\boldsymbol{k}\times\boldsymbol{k})=0+\boldsymbol{j}-3\boldsymbol{k}-\boldsymbol{i}-3\boldsymbol{j}+0=-\boldsymbol{i}-2\boldsymbol{j}-3\boldsymbol{k}$$

Next, by properties of scalar (or dot) product,

$\boldsymbol{a}\cdot(\boldsymbol{b}+\boldsymbol{c})=\boldsymbol{a}\cdot\boldsymbol{b}+\boldsymbol{a}\cdot\boldsymbol{c}$**,** $\boldsymbol{i}\cdot\boldsymbol{j}=\boldsymbol{i}\cdot\boldsymbol{k}=\boldsymbol{j}\cdot\boldsymbol{k}=0$**,** $\boldsymbol{i}\cdot\boldsymbol{i}=\boldsymbol{j}\cdot\boldsymbol{j}=\boldsymbol{k}\cdot\boldsymbol{k}=1$**.**

If vectors are given by their coordinates, $\boldsymbol{a}=(a_x,a_y,a_z)$ **and** $\boldsymbol{b}=(b_x,b_y,b_z)$**, then**

$$\boldsymbol{a}\cdot\boldsymbol{b}=\left(\boldsymbol{a_x i}+\boldsymbol{a_y j}+\boldsymbol{a_z k}\right)\left(\boldsymbol{b_x i}+\boldsymbol{b_y j}+\boldsymbol{b_z k}\right)=\boldsymbol{a_x b_x}+\boldsymbol{a_y b_y}+\boldsymbol{a_z b_z}.$$

These formulae allow to simplify

$$(2\boldsymbol{i}-3\boldsymbol{j})\cdot\ [(\boldsymbol{i}+\boldsymbol{j}-\boldsymbol{k})\times(3\boldsymbol{i}-\boldsymbol{k})]=(2\boldsymbol{i}-3\boldsymbol{j})\cdot(-\boldsymbol{i}-2\boldsymbol{j}-3\boldsymbol{k})=$$

$$=\mathbf{2}\cdot(-\mathbf{1})+(-\mathbf{3})\cdot(-\mathbf{2})=-\mathbf{2}+\mathbf{6}=\mathbf{4}.$$

Answer: 4.