Answer on Question #52237 – Math – Vector Calculus

What are vectors that are not parallel to the same line, called?

scalar

collinear vectors

non-collinear vectors

vectors

Answer: non-collinear vectors.

7 Find the vector product axb. If a = 2i + 3j + 4k and b = 5i - 2j + k

11i + 18j - 19k

2j + 3k

5i - 6j + 7k

4i - 6j +11k

Solution

$$\vec{a}x\vec{b} = \begin{vmatrix} i & j & k \\ 2 & 3 & 4 \\ 5 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 3 & 4 \\ -2 & 1 \end{vmatrix} i - \begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} j + \begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix} k =$$

$$= i\big(3 \cdot 1 - 4(-2)\big) + j(4 \cdot 5 - 1 \cdot 2) + k(2 \cdot (-2) - 3 \cdot 5) = 11i + 18j - 19k.$$

Answer: 11i + 18j - 19k.

8 A ...... line AB occurs when the point A is fixed.

free vector

position vector

force

null vector

Answer: position vector.

9 A north-easterly wind of 20 knots is a ...... quantity.

scalar

weight

vector

distance

Answer: vector.

10 Given that a = 5i +2j - k and b = I - 3j + k. Find (a + b) x (a - b).

2i - 12j - 34k

2i + 12j + 34k

2i -3j + 12j

2i + 2k

Solution

Method 1 (straight-forward calculation)

$$(\vec{a}+\vec{b}) = 5i + 2j - k + i - 3j + k = 6i - j.$$

$$(\vec{a}-\vec{b}) = 5i + 2j - k - i + 3j - k = 4i + 5j - 2k.$$

$$(\vec{a}+\vec{b})\times(\vec{a}-\vec{b}) = \begin{vmatrix} i & j & k \\ 6 & -1 & 0 \\ 4 & 5 & -2 \end{vmatrix} = \begin{vmatrix} -1 & 0 \\ 5 & -2 \end{vmatrix} i - \begin{vmatrix} 6 & 0 \\ 4 & -2 \end{vmatrix} j + \begin{vmatrix} 6 & -1 \\ 4 & 5 \end{vmatrix} k$$
$$= (-1\cdot(-2) - 5\cdot 0)i - (6\cdot(-2) - 4\cdot 0)j + (6\cdot 5 - 4\cdot(-1))k = 2i + 12j + 34k.$$

Method 2 (application of cross product properties)

The following properties of the cross product will be used:

$\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$; $\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b} + \vec{a}\times\vec{c}$; $\vec{a}\times\vec{a} = \vec{b}\times\vec{b} = \vec{0}$;

$$(\lambda\vec{a})\times\vec{b} = \vec{a}\times(\lambda\vec{b}) = \lambda(\vec{a}\times\vec{b})$$

$$\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \begin{vmatrix} a_y & a_z \\ b_y & b_z \end{vmatrix} i - \begin{vmatrix} a_x & a_z \\ b_x & b_z \end{vmatrix} j + \begin{vmatrix} a_x & a_y \\ b_x & b_y \end{vmatrix} k =$$
$$= (a_y b_z - b_y a_z)i + (a_z b_x - a_x b_z)j + (a_x b_y - a_y b_x)k$$

Simplify

$$(\vec{a}+\vec{b})\times(\vec{a}-\vec{b}) = (\vec{a}\times\vec{a}) + (\vec{a}\times(-\vec{b})) + (\vec{b}\times\vec{a}) + (\vec{b}\times(-\vec{b}))$$
$$= (\vec{a}\times\vec{a}) - (\vec{a}\times\vec{b}) + (\vec{b}\times\vec{a}) - (\vec{b}\times\vec{b}) = \vec{0} - (\vec{a}\times\vec{b}) - (\vec{a}\times\vec{b}) - \vec{0} = -2(\vec{a}\times\vec{b})$$
$$= -2\begin{vmatrix} i & j & k \\ 5 & 2 & -1 \\ 1 & -3 & 1 \end{vmatrix} = -2\left(\begin{vmatrix} 2 & -1 \\ -3 & 1 \end{vmatrix} i - \begin{vmatrix} 5 & -1 \\ 1 & 1 \end{vmatrix} j + \begin{vmatrix} 5 & 2 \\ 1 & -3 \end{vmatrix} k\right)$$
$$= -2\left((2\cdot 1 - (-3)\cdot(-1))i - (5\cdot 1 - 1\cdot(-1))j + (5\cdot(-3) - 1\cdot 2)k\right)$$
$$= -2(-i - 6j - 17k) = 2i + 12j + 34k$$

Answer: 2i+12j+34k.