Answer on Question #52226 – Math - Multivariable Calculus

Given:

$f(x,y) = \sin^2 x \cdot \cos y + xy^2$

Solution:

Let's compute the partial derivative of function f(x,y) with respect to x, with y held constant:

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}\left(\sin^2 x \cdot \cos y + xy^2\right) = \frac{\partial}{\partial x}\left(\sin^2 x \cdot \cos y\right) + \frac{\partial}{\partial x}\left(xy^2\right) = \cos y \frac{\partial}{\partial x}\left(\sin^2 x\right) + y^2 \frac{\partial}{\partial x}(x);$$

$$\frac{\partial f}{\partial x} = \cos y \frac{\partial}{\partial x}\left(\sin^2 x\right) + y^2 \frac{\partial}{\partial x}(x) = \cos y \cdot 2\sin x \cdot \cos x + y^2 = 2\sin x \cdot \cos x \cdot \cos y + y^2;$$

$$\frac{\partial f}{\partial x} = \sin 2x \cdot \cos y + y^2.$$

The rules work the same way here as it does with functions of one variable:

$\frac{\partial}{\partial x}(g(x,y) + h(x,y)) = \frac{\partial g(x,y)}{\partial x} + \frac{\partial h(x,y)}{\partial x}$;

$\frac{\partial}{\partial x}(A(y) r(x,y)) = A(y) \frac{\partial r(x,y)}{\partial x}$, where $A(y)$ is constant with respect to x;

$\frac{\partial}{\partial x}(\sin x) = \cos x$;

$\frac{\partial}{\partial x}(x^n) = nx^{n-1}$, where n is integer.

The chain rule is $\frac{\partial}{\partial x} f(g(x)) = \left.\frac{\partial f(t)}{\partial t}\right|_{t=g(x)} \frac{\partial g}{\partial x} = f'(g(x)) \cdot g'(x)$.

In this question $\sin^2 x = f(g(x))$ is composite function, where $f(x) = x^2$, $g(x) = \sin x$.