Answer on Question #52190 – Math – Calculus

1. If y = arc sin x, then
dy/dx
is

Solution

Consider $y = \arcsin x$, $-1 \le x \le 1$, $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, $\cos(y) \ge 0$, where $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

Besides, by Pythagorean theorem, $\cos^2 y + \sin^2 y = 1$.

Function $y = \arcsin x$ is the inverse of $y = \sin x$. It means that if $y = f(x)$, $f(x) = \arcsin x$, then $x = \sin y$, $x = g(y)$, $g(y) = \sin y$.

It is known that $g'(y) = (\sin y)' = \cos y$.

The derivative of the inverse function is given by

$$g'(y) = \frac{1}{f'(x)},$$

hence

$$f'(x) = \frac{1}{g'(y)} = \frac{1}{(\sin y)'} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}}.$$

The derivative of the function is

$$\frac{dy}{dx} = \frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}.$$

Answer: $\frac{1}{\sqrt{1-x^2}}$.