Answer on Question #52167 – Math – Multivariable Calculus

Find the total mass of a plate in the shape of the region bounded by y=x^(-1) and y=0 for 1<x<4 and the mass density is f(x,y)=y/x .

Solution

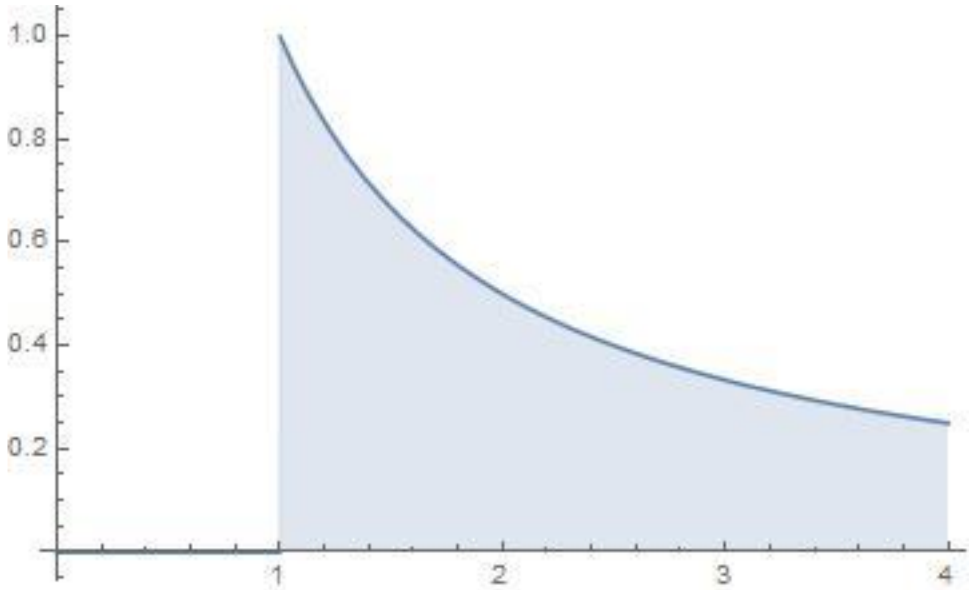
The total mass of a plate is given by

$$m = \iint_S f(x,y)dxdy$$

where f(x,y) – mass density; S – body (plate), its mass to be found.

$$S:\ y = \frac{1}{x}; y = 0;\ \ 1 < x < 4.$$

$$0 \le y \le \frac{1}{x};\quad 1 < x < 4;$$



$$m = \iint\limits_S f(x,y)dxdy =$$

$$= \int\limits_1^4 dx \int\limits_0^{\frac{1}{x}} \frac{y}{x} dy = \int_1^4 \frac{1}{x}\left(\int_0^{\frac{1}{x}} ydy\right)dx = \int_1^4 \frac{1}{x}\left(\frac{y^2}{2}\bigg|_0^{\frac{1}{x}}\right)dx =$$

$$= \int_1^4 \frac{1}{2x}\left(\frac{1}{x^2} - 0\right)dx = \int_1^4 \frac{1}{2x^3}dx = \frac{1}{2}\int_1^4 x^{-3}dx = \frac{1}{2}\left(\frac{x^{-2}}{-2}\bigg|_1^4\right) = -\frac{1}{4x^2}\bigg|_1^4 =$$

$$= -\frac{1}{4\cdot 4^2} + \frac{1}{4} = \frac{1}{4}\left(1 - \frac{1}{16}\right) = \frac{15}{64}.$$

To evaluate the total mass, Newton-Leibnitz formula was applied to two definite integrals:

$$\frac{1}{x}\int\limits_0^{\frac{1}{x}} ydy = \frac{y^2}{2x}\bigg|\begin{matrix}\frac{1}{x}\\0\end{matrix} = \frac{1}{2x^3} - 0 = \frac{1}{2x^3}.$$

$$m = \int\limits_1^4 \frac{1}{2x^3}dx = -\frac{1}{4x^2}\bigg|\begin{matrix}4\\1\end{matrix} = -\frac{1}{4*16} + \frac{1}{4} = \frac{15}{64}.$$

Answer: $m = \frac{15}{64}$.