Answer on Question #52107 - Math - Calculus

1)Find the directional derivative of $f(x,y,z) = xy^2 - z^2$in the direction of v=(1,-2,2) at (2,1,3).

ANSWER:

Solution:

Using the chain rule, we compute the partial derivatives of f:

$f_x = y^2$

$f_y = 2xy$

$f_z = -2z.$

At the point (2,1,3), these become

$f_x(2,1,3) = 1^2 = 1$

$f_y(2,1,3) = 2 \cdot 2 \cdot 1 = 4$

$f_z(2,1,3) = -2 \cdot 3 = -6.$

vector $v = 1i - 2j + 2k$, $|v| = \sqrt{1^2 + (-2)^2 + 2^2} = \sqrt{5}.$

Normalising v, we have:

$$u = \frac{v}{|v|} = \frac{1}{\sqrt{5}}i - \frac{2}{\sqrt{5}}j + \frac{2}{\sqrt{5}}k.$$

Thus the directional derivative is

$$\boldsymbol{D_u f(2,1,3)} = \nabla f(2,1,3) \cdot u = 1 \cdot \frac{1}{\sqrt{5}} - 4 \cdot \frac{2}{\sqrt{5}} - 6 \cdot \frac{2}{\sqrt{5}} = \frac{1}{\sqrt{5}} - \frac{8}{\sqrt{5}} - \frac{12}{\sqrt{5}} = -\boldsymbol{\frac{19}{\sqrt{5}}}.$$

2)Suppose f(1,0,0)=-3,fx(1,0,0)=-2,fy(1,0,0)=4 and fz(1,0,0)=2 . Use linear approximation to $estimate\ f(1.02, 0.01, -0.03)$.

ANSWER:

The above linear approximation at $(x,y,z) = (1.02, 0.01, -0.03)$ is

$$L(x,y,z)=\mathbf{f}(1,0,0)+D\mathbf{f}(1,0,0)(x-1,y,z)$$

$$L(1.02, 0.01, -0.03) = -3 + f_x(1,0,0)(x-1) + f_y(1,0,0)y + f_z(1,0,0)z$$

$$= -3 + (-2)(1.02 - 1) + 4 \cdot 0.01 + 2 \cdot (-0.03) = \mathbf{-3.03}$$

3) Find an equation of the tangent plane at the point(4,1) of the surface $z = x^2 + y^{-2}$.

ANSWER:

The given surface is the level surface of

$F(x,y,z) = z - x^2 - y^{-2}$

defined by the equation F(x, y, z) = 0.

$F_x = -2x$

$F_y = 2y^{-3}$

$F_z = 1.$

Recall that the tangent plane to a

level surface is orthogonal to the gradient vector of F, and so we compute

$\nabla \mathrm{F}(x,y,z) = (-2x, 2y^{-3}, 1).$

At the point $(4, 1, z)$, we have

$\nabla \mathrm{F}(4,1,z) = (-8,2,1)$

and so the tangent plane is given by the equation

$-8(x-4) + 2(y-1) + 1(z-z) = 0.$

$-8x + 32 + 2y - 2 + 0 \cdot z = 0.$

$-8x + 2y + 0 \cdot z + 30 = 0.$

which simplifies to

$\boldsymbol{-4x + y + 0 \cdot z + 15 = 0}.$