Answer on Question #52000-Math-Calculus

Find the directional derivative of

$$f(x,y) = \begin{cases} \dfrac{2xy^2}{x^2 + y^4}, (x,y) \neq (0,0) \\ 0, (x,y) = (0,0) \end{cases}$$

at $(0,0)$ in the direction $\theta = \frac{\pi}{3}$.

Solution

$$\vec{\nabla} f = \langle \frac{2y^2(x^2 + y^4) - 2xy^2 \cdot 2x}{(x^2 + y^4)^2}, \frac{4xy(x^2 + y^4) - 2xy^2 \cdot 4y^3}{(x^2 + y^4)^2} \rangle = \langle \frac{2y^2(y^4 - x^2)}{(x^2 + y^4)^2}, \frac{4xy(x^2 - y^4)}{(x^2 + y^4)^2} \rangle$$

We can express this as

$$\vec{\nabla} f = f(x,y) \cdot \langle \frac{(y^4 - x^2)}{x(x^2 + y^4)}, -\frac{2(y^4 - x^2)}{y(x^2 + y^4)} \rangle.$$

At $(x,y) = (0,0)$ $f(x,y) = 0$. Thus,

$$\vec{\nabla} f(0,0) = f(0,0) \cdot \langle \frac{(y^4 - x^2)}{x(x^2 + y^4)}, -\frac{2(y^4 - x^2)}{y(x^2 + y^4)} \rangle_{(0,0)} = 0.$$

$$\vec{u} = \langle \cos\frac{\pi}{3}, \sin\frac{\pi}{3} \rangle = \frac{1}{2} \langle 1, \sqrt{3} \rangle.$$

$$D_{\vec{u}} f(0,0) = \vec{\nabla} f(0,0) \cdot \vec{u} = \frac{1}{2}\left(1 \cdot 0 + \sqrt{3} \cdot 0\right) = 0.$$

Answer: 0.