Answer on Question #51999-Math-Calculus

b) Compute the Jacobian matrices using the chain rule for $z = u^2 + v^2$ where (3)

$$u = 2x + 7, v = 3x + y + 7.$$

Solution

Let $z = f(u,v) = u^2 + v^2$ and $g(u,v) = (2x+7, 3x+y+7)$.

$$Df = (f_u, f_v) = (2u, 2v).$$

$$Dg = \begin{pmatrix} u_x & u_y \\ v_x & v_y \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix}.$$

So,

$$F(f \circ g) = Df \cdot Dg = (2u, 2v)\begin{pmatrix} 2 & 0 \\ 3 & 1 \end{pmatrix} = (4u + 6v, 2v).$$

Answer: $(\mathbf{4u + 6v, 2v})$.

c) if

$$f(x,y) = \begin{cases} \dfrac{4x^2y^2}{x^2+y^2}, (x,y) \neq (0,0) \\ 0, (x,y) = (0,0) \end{cases}$$

Then show

$$f_{xy}(0,0) = f_{yx}(0,0)$$

Solution

$$f_X(x,y) = \frac{8xy^2(x^2+y^2) - 4x^2y^2 \cdot 2x}{(x^2+y^2)^2} = \frac{8xy^4}{(x^2+y^2)^2}.$$

$$f_{xy} = \frac{4 \cdot 8xy^3(x^2+y^2)^2 - 8xy^4 \cdot 2(x^2+y^2)2y}{(x^2+y^2)^4} = \frac{32x^3y^3}{(x^2+y^2)^3}.$$

$$f_y(x,y) = \frac{8yx^2(x^2+y^2) - 4x^2y^2 \cdot 2y}{(x^2+y^2)^2} = \frac{8yx^4}{(x^2+y^2)^2}.$$

$$f_{xy} = \frac{4 \cdot 8yx^3(x^2+y^2)^2 - 8yx^4 \cdot 2(x^2+y^2)2x}{(x^2+y^2)^4} = \frac{32x^3y^3}{(x^2+y^2)^3}.$$

Thus,

$$f_{xy} = f_{yx} \; for \; all \; (x,y).$$

Therefore

$$f_{xy}(0,0) = f_{yx}(0,0).$$