Answer on Question #51811 – Math – Trigonometry

Deduce the identity of :
$1-2\sin^2 A$

A *sin 2A*
B *tan 2A*
C *cos 2A*
D *cos A*

Solution

Method 1

This method uses ideas of Trigonometry.

Recall formulas

$$\cos^2 A + \sin^2 A = 1$$

and $\cos(2A) = \cos^2 A - \sin^2 A$, which follows from

$$\cos(A + B) = \cos A \cdot \cos B - \sin A \cdot \sin B.$$

Then

$$1 - 2\sin^2 A = (\cos^2 A + \sin^2 A) - 2\sin^2 A =$$

$$= \cos^2 A + (\sin^2 A - 2\sin^2 A) = \cos^2 A - \sin^2 A = \cos(2A)$$

Method 2

This method uses ideas of Complex Analysis.

According to Euler's formula $\sin\alpha = \left(e^{i\alpha} - e^{-i\alpha}\right)/2$,

$$1-2\sin^2\alpha = 1-2\cdot\left(\frac{\exp[i\alpha]-\exp[-i\alpha]}{2i}\right)^2 = 1-2\frac{\left(\exp[2i\alpha]-2\exp[i\alpha]\exp[-i\alpha]+\exp[-2i\alpha]\right)}{4i^2} =$$

$$= 1+\frac{1}{2}\left(\exp[2i\alpha]-2+\exp[-2i\alpha]\right) = \frac{\left(\exp[2i\alpha]+\exp[-2i\alpha]\right)}{2} = \cos 2\alpha$$

Answer: C cos (*2A*)