# Answer on Question #51711 - Math - Vector Calculus

$\nabla(1/r) =?$,  $\nabla^2(1/r) =?$

where

$\vec{r} = x\vec{\imath} + y\vec{\jmath} + z\vec{k}$

**Solution**

$\nabla(1/r)$:

$$\nabla\left(\frac{1}{r}\right) = \left[\frac{\partial}{\partial x}\left(\frac{1}{r}\right)\right]\vec{\imath} + \left[\frac{\partial}{\partial y}\left(\frac{1}{r}\right)\right]\vec{\jmath} + \left[\frac{\partial}{\partial z}\left(\frac{1}{r}\right)\right]\vec{k}$$

Since $r = \sqrt{x^2 + y^2 + z^2}$, we obtain

$$\frac{\partial}{\partial x}\left(\frac{1}{r}\right) = \frac{\partial}{\partial x}\left(\frac{1}{\sqrt{x^2 + y^2 + z^2}}\right) = -\frac{\frac{\partial}{\partial x}(x^2 + y^2 + z^2)}{2\left(\sqrt{x^2 + y^2 + z^2}\right)^3} =$$

$$= -\frac{2x}{2\left(\sqrt{x^2 + y^2 + z^2}\right)^3} = -\frac{x}{r^3}$$

Similarly

$$\frac{\partial}{\partial y}\left(\frac{1}{r}\right) = -\frac{y}{r^3}$$

$$\frac{\partial}{\partial z}\left(\frac{1}{r}\right) = -\frac{z}{r^3}$$

Substituting these into the first equation we obtain

$$\nabla\left(\frac{1}{r}\right) = \left[\frac{\partial}{\partial x}\left(\frac{1}{r}\right)\right]\vec{\imath} + \left[\frac{\partial}{\partial y}\left(\frac{1}{r}\right)\right]\vec{\jmath} + \left[\frac{\partial}{\partial z}\left(\frac{1}{r}\right)\right]\vec{k} = -\frac{x}{r^3}\vec{\imath} - \frac{y}{r^3}\vec{\jmath} - \frac{z}{r^3}\vec{k} = -\frac{\vec{r}}{r^3}$$

$\nabla^2(1/r)$:

$$\nabla^2\left(\frac{1}{r}\right) = \frac{\partial^2}{\partial x^2}\left(\frac{1}{r}\right) + \frac{\partial^2}{\partial y^2}\left(\frac{1}{r}\right) + \frac{\partial^2}{\partial z^2}\left(\frac{1}{r}\right)$$

$$\frac{\partial^2}{\partial x^2}\left(\frac{1}{r}\right) = \frac{\partial}{\partial x}\left(-\frac{x}{r^3}\right) = -\frac{\frac{\partial}{\partial x}(x)}{r^3} - x\frac{\partial}{\partial x}\left(\frac{1}{r^3}\right) =$$

$$= -\frac{1}{r^3} - x\frac{\partial}{\partial x}\left(\frac{1}{\left(\sqrt{x^2 + y^2 + z^2}\right)^3}\right) =$$

$$= -\frac{1}{r^3} + x\frac{3}{2}\frac{\frac{\partial}{\partial x}(x^2+y^2+z^2)}{\left(\sqrt{x^2+y^2+z^2}\right)^5} = -\frac{1}{r^3} + \frac{3x^2}{\left(\sqrt{x^2+y^2+z^2}\right)^5} = -\frac{1}{r^3} + \frac{3x^2}{r^5}$$

Similarly

$$\frac{\partial^2}{\partial y^2}\left(\frac{1}{r}\right) = -\frac{1}{r^3} + \frac{3y^2}{r^5}$$

$$\frac{\partial^2}{\partial z^2}\left(\frac{1}{r}\right) = -\frac{1}{r^3} + \frac{3z^2}{r^5}$$

Therefore

$$\nabla^2\left(\frac{1}{r}\right) = \frac{\partial^2}{\partial x^2}\left(\frac{1}{r}\right) + \frac{\partial^2}{\partial y^2}\left(\frac{1}{r}\right) + \frac{\partial^2}{\partial z^2}\left(\frac{1}{r}\right) = -\frac{1}{r^3} + \frac{3x^2}{r^5} - \frac{1}{r^3} + \frac{3y^2}{r^5} - \frac{1}{r^3} + \frac{3z^2}{r^5} =$$

$$= -\frac{3}{r^3} + 3\frac{x^2+y^2+z^2}{r^5} = -\frac{3}{r^3} + 3\frac{r^2}{r^5} = -\frac{3}{r^3} + \frac{3}{r^3} = 0$$

**Answer**: $\nabla\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}, \;\; \nabla^2\left(\frac{1}{r}\right) = 0.$