

### Answer on Question #51704 – Math – Real Analysis

**Question.** If the co-domain does not contain all the elements of range, can it be a function, like co-domain  $\{3, 4, 5, 6, 7\}$  range  $\{3, 4, 7\}$ ? Normally we know range is a subset of co-domain. But here it does not. So is it possible?

**Answer.** By definition, a *function*  $f : X \rightarrow Y$  is the correspondence which associates to each  $x \in X$  a *unique* element from  $Y$  denoted by  $f(x)$ .

The set  $X$  is then called the *domain* of  $f$ , the set  $Y$  is said to be the *co-domain*, and the set  $f(X) = \{f(x) \in Y \mid x \in X\}$  is the *range* of  $f$ .

The above definition **does not require** that *the range*  $f(X)$  *coincides with all the co-domain*  $Y$ . So in general, the range of the function can be a proper subset of the co-domain:

$$f(X) \subsetneq Y.$$

The functions for which the range coincides with the co-domain  $f(X) = Y$  are called *surjective*.

For example, the functions  $\sin, \cos : \mathbb{R} \rightarrow \mathbb{R}$  are not surjective have the same range  $[-1, 1]$ .

Also notice that every function  $f : X \rightarrow Y$  induces a surjective function  $\hat{f} : X \rightarrow f(X)$  defined by  $\hat{f}(x) = f(x)$ . In other words, we can always “replace” co-domain with the range to get a surjective function.

Summarize all that is said above: in general, the range of the function can be a proper subset of the co-domain.