Answer on Question #51699 – Math – Algebra

if a function is neither one-one nor onto , can we say it's a function? if yes , then which function. like this example f:A to B y=f(x)=x^2 +3 A={-1,1,2,3} B={4,5,6,7}

THIS IS NEITHER ONE-ONE NOR ONTO. SO , if it is a function, so what is the name of this ?

Solution

Main parts of function: the input; the relationship, the output.

Definition 1. *A function* $\boldsymbol{f}$ *from a set* $\boldsymbol{A}$ *to a set* $\boldsymbol{B}$ *relates each element of* ***A*** *with exactly one element of* $\boldsymbol{B}$ *(the same set* ***A=B*** *is possible).*

In other words, every element in ***A*** is related to some element in ***B*** (But some elements of ***B*** might not be related to at all, which is fine).

Besides, a function is single valued, that is, function will not give back two or more results for the same input (for example, "***f(2)=5, f(2)=6***" is not correct).

The one-to-many case "$\boldsymbol{f(a_1) = b_1, f(a_1) = b_2}$ " is not allowed, but

many-to-one case "$\boldsymbol{f(a_1) = b_1, f(a_2) = b_1}$" is allowed.

Definition 2. *A function* ***f: A*** → ***B*** *is called* ***onto*** *if for all* $\boldsymbol{b}$ *in* $\boldsymbol{B}$ *there is an* $\boldsymbol{a}$ *in* $\boldsymbol{A}$ *such that* $\boldsymbol{f(a) = b}$.

Then all elements in $\boldsymbol{B}$ are used.

Definition 3. *A function* ***f: A*** → ***B*** *is called* ***one-to-one*** *if whenever* $\boldsymbol{f(a_1) = f(a_2)}$ *then* $\boldsymbol{a_1 = a_2}$.

No element of B is the image of more than one element in A.

The neither one-to-one nor onto function (in other words neither injective nor surjective function) does not have special name. It is a function in general.

Example 1. *If **f: A** → **B**, where **A**=ℝ, **B**=ℝ, then* $\boldsymbol{f(x) = x^2}$ *is an example of neither one-to-one nor onto function. It is not onto, since the image of* $\boldsymbol{f(x)}$ *is* $[\mathbf{0}; +\infty)$***,** and not one-to-one, since* $\boldsymbol{f(-1) = f(1)}$***.***

Example 2. ***A function in this question*** *(**f: A** → **B,** where **A**={-1,1,2,3} , **B**={4,5,6,7}, **y=f(x)**, **f(x)=x^2 +3**)* ***is not defined*** *at point **3,** because a function has only one relationship for each input value from **A**, but y(3)=3^2+3=12 and 12 is not element of **B**. In other words, element 3 from **A** is not assigned to any element of **B**.*

Example 3. *If we take **y=f(x)**, **f(x)=x^2 +3**, **A**={-1,1,2,3}, **B**={4,5,,7,12}, then function **f: A** → **B** is well defined, it is onto function, but not one-to-one (here* $\boldsymbol{f(-1) = f(1)}$*)**.***

Example 4. *By means of the least squares method we can suggest **another function** y=f(x), for example, y=4.25+0.541667*x*+0.25*x^2*-0.041667*x^3*, which approximately represents **f: A** → **B**, where A={-1,1,2,3}, B={4,5,6,7}.*