Answer on Question #51698 – Math – Combinatorics | Number Theory

Let p be a prime number. If p divides a^2, prove that p divides a,where a is a positive integer.

Solution

Let p divide a^2. Assume that p doesn't divide a.

Since p divides a^2, then there exists integer k such that $a^2 = pk$. Hence we obtain $a = \frac{pk}{a}$.

Since a is an positive integer, then $\frac{pk}{a}$ is an positive integer. Since p is a prime number then it has only two divisors: 1 and p. Due to our assumption p doesn't divide a, therefore $GCD(p,a) = 1$. Hence $\frac{k}{a}$ is a positive integer. Assume that $t = \frac{k}{a}$, then t is a positive integer.

Therefore $a = \frac{pk}{a} = pt$, where t is a positive integer, but this means that p divides a. So, we come to a contradiction to our assumption.

Thus, p divides a.