Answer on Question #51697, Math, Calculus:

$$\frac{x}{(x+1)(x^3+1)} = -\frac{1}{3(x+1)^2} + \frac{1}{3(x^2-x+1)}$$

Solution

Decompose this partial fraction:

$$\frac{x}{(x+1)(x^3+1)}$$

One root of the x^3+1 polynomial is -1. To find the factoring, divide the polynomial by $x+1$:

$$\begin{array}{r|l} x^3 \qquad\quad +1 & x+1 \\ \cline{2-2} x^3+x^2 \qquad\quad & x^2-x+1 \\ \cline{1-1} -x^2 \quad +1 & \\ -x^2-x \quad & \\ \cline{1-1} x+1 & \\ x+1 & \\ \cline{1-1} 0 & \end{array}$$

i.e. $x^3+1 = (x+1)(x^2-x+1)$
Therefore:

$$\frac{x}{(x+1)(x^3+1)} = \frac{x}{(x+1)(x+1)(x^2-x+1)} = \frac{x}{(x+1)^2(x^2-x+1)}$$

Represent the task fraction as:

$$\frac{x}{(x+1)^2(x^2-x+1)} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{Cx+D}{x^2-x+1} =$$

$$= \frac{A(x+1)(x^2-x+1) + B(x^2-x+1) + Cx(x+1)^2 + D(x+1)^2}{(x+1)^2(x^2-x+1)} =$$

$$= \frac{A(x^3+1) + B(x^2-x+1) + C(x^3+2x^2+x) + D(x^2+2x+1)}{(x+1)^2(x^2-x+1)}$$

Equate the rates of equal powers:

$$x^3\colon A+C=0$$
$$x^2\colon B+2C+D=0$$
$$x^1\colon -B+C+2D=1$$
$$x^0\colon A+B+D=0$$

The first equation is equal to $A=-C$
Sum of the third and the forth equations gives:

$$A+C+3D=1 \Rightarrow D=\frac{1}{3}$$

Sum of the second and the third equations gives:

$$3C+3D=1 \Rightarrow C=\frac{1}{3}-D=0$$

From the third equation:

$$B = 2D - 1 = -\frac{1}{3}$$

Finally:

$$\frac{x}{(x+1)(x^3+1)} = -\frac{1}{3(x+1)^2} + \frac{1}{3(x^2-x+1)}$$