Answer on Question #51693 – Math – Calculus

Question

decomposition of this partial fraction x-1/[(x-2)(x^2 +3)] ≡ [A/(x-2)]+[Bx+C/(x^2 +3)] ? why we use Bx+C here ??? and what will happen when it is (x^3 +3) or (x^4 +3) instead of (x^2 +3)

Solution

1. $$\frac{x-1}{(x-2)(x^2+3)} \equiv \frac{A}{x-2} + \frac{Bx+C}{x^2+3}$$

$$x-1 = A(x^2+3) + (Bx+C)(x-2)$$

$$x=2 \quad 2-1 = A(2^2+3) \Rightarrow A = \frac{1}{7}$$

$$x-1 = Ax^2 + 3A + Bx^2 - 2Bx + Cx - 2C$$

$$x-1 = (A+B)x^2 + (-2B+C)x + (3A-2C)$$

$$\begin{cases} A+B=0 \\ 3A-2C=-1 \Rightarrow \\ -2B+C=1 \end{cases} \quad \begin{matrix} B=-A=-\frac{1}{7} \\ \\ C=1+2B=\frac{5}{7} \end{matrix}$$

$$\frac{x-1}{(x-2)(x^2+3)} \equiv \frac{\frac{1}{7}}{x-2} + \frac{(-\frac{1}{7}x+\frac{5}{7})}{x^2+3}$$

2. If the denominator of your rational expression is $ax+b$, so we add $\frac{A}{ax+b}$ to the partial fraction decomposition, where A is a number.

If the denominator of your rational expression is $(ax+b)^2$, so we add $\frac{A}{ax+b} + \frac{B}{(ax+b)^2}$ to the partial fraction decomposition, where $A and B$ are numbers.

If the denominator of your rational expression is ax^2+bx+c and the discriminant

$D = b^2 - 4ac < 0$, then we add $\frac{Ax+B}{ax^2+bx+c}$ to the partial fraction decomposition. We use $Ax+B$ here because discriminant of ax^2+bx+c is less than 0, so equation ax^2+bx+c =0 doesn`t have real roots.

If the denominator of your rational expression is ax^2+bx+c and the discriminant

$D = b^2 - 4ac > 0$, then $ax^2 + bx + c = a(x - x_1)(x - x_2)$ and we add $\frac{E}{x-x_1} + \frac{F}{x-x_2}$ to the partial fraction decomposition.

3. When we put (x^3+3) instead of (x^2+3) , we get

$$\frac{x-1}{(x-2)(x^3+3)} = \frac{x-1}{(x-2)\left(x+\sqrt[3]{3}\right)\left(x^2 - x\sqrt[3]{3} + \sqrt[3]{9}\right)} = \frac{A}{x-2} + \frac{B}{x+\sqrt[3]{3}} + \frac{Cx+D}{x^2 - x\sqrt[3]{3} + \sqrt[3]{9}}$$

When we put (x^4+3) instead of (x^2+3) , we get

$$\frac{x-1}{(x-2)(x^4+3)} = \frac{x-1}{(x-2)(x^4+2x^2+3-2x^2)} = \frac{x-1}{(x-2)\left(x^2+1-x\sqrt{2}\right)\left(x^2+1+x\sqrt{2}\right)}$$
$$= \frac{A}{x-2} + \frac{Bx+C}{x^2+1-x\sqrt{2}} + \frac{Dx+E}{x^2+1+x\sqrt{2}}$$